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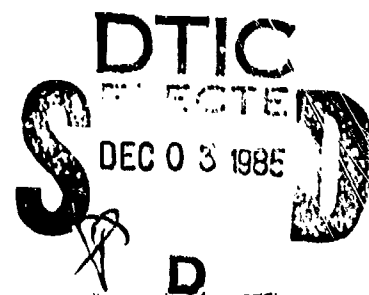
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AERODYNAMIC AND FLIGHT DYNAMIC CHARACTERISTICS OF THE NEW FAMILY OF 5.56MM NATO AMMUNITION

Robert L. McCoy

October 1985



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The M855/SS-109 projectiles show limit-cycle yaw levels growing to approximately six degrees at 800 metres range. *Originator-supplied keywaying*

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I. INTRODUCTION

On 24 March 1983, the author attended a meeting with personnel of the Squad Automatic Weapon (SAW) Project Office at Dover, New Jersey. The subject of the meeting was the unacceptably high lot rejection rates of early production 5.56mm M855 (Ball) and M856 (Tracer) ammunition manufactured at Lake City Army Ammunition Plant (LCAAP). The rejected lots failed to meet the accuracy specification, and LCAAP had indicated to the SAW Project Office that they believed the government-furnished test barrels might be contributing to the problem.

The result of the meeting was a joint recommendation, by the Ballistic Research Laboratory (BRL) and the SAW Project Office, to conduct a three-part test at the BRL free-flight range facility. The first part of the test was to be an accuracy check, using the Kart-manufactured barrels supplied to LCAAP. The accuracy test was to include rejected lots of M855/M856, control lots of the Belgian (FNB) counterpart SS-109/L110 ammunitions, and handloaded ammunition using 52 grain Sierra Benchrest bullets, in both Lake City cartridge cases, and commercial match grade cases. All accuracy firing was to be done at 100 yards, in the BRL Aerodynamics Range.¹ The second part of the test consisted of aeroballistic range firings to determine the aerodynamic and flight characteristics of the LCAAP and FNB ammunitions, using down-loaded propellant charges to simulate ranges out to 800 metres. The third test phase was a real-range determination of striking velocity and limit-cycle yaw for the four ammunition types, using the limit-cycle test equipment in the BRL Transonic Range.

Test materiel and funding were provided to BRL by the SAW Project Office, and the first phase of testing began on 31 May 1983. The accuracy tests were completed on 27 June 1983, and the second part of the test schedule was conducted in September-October 1983. The third phase, limit-cycle yaw testing was conducted in March 1984. This report covers the results from all three phases of the BRL tests.

II. TEST MATERIEL AND PROCEDURE

Two of the Kart accuracy barrels, chambered and threaded to fit a Remington M700 action, were supplied to the BRL by the SAW Office. The Kart barrel serial numbers were 014 and 018, and both barrels had previously been in use at LCAAP; these two barrels were among those suspected by LCAAP personnel of contributing to the accuracy problem. One goal of the Phase I testing was to determine which of these two barrels was the more accurate, and select it for the remainder of the tests.

1. W. F. Braun, "The Free Flight Aerodynamics Range," Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, BRL Report No. 1048, August 1958. (AD 202249)

Figure 1 is a photograph of the accuracy test set-up. The barrelled action is mounted in a Frankford rest, with a Leupold M8, 16X scope to check return-to-battery alignment between shots. By using a systematic procedure for returning the recoil cradle to battery, it was determined that shot-to-shot scope alignment within $\pm 1/16"$ at 100 yards range could be maintained.

The U.S. lot acceptance accuracy specification for 5.56mm NATO Ball ammunition is as follows: the mean radius of each group of ten shots shall not exceed 2 inches at 200 yards. Tracer ammunition must meet a less stringent requirement; less than 4 inch mean radius at the same range. For the BRL 100 yard indoor firings, the required accuracy criteria translate to 1-inch mean radius (MR) for the M855 and SS-109, and 2-inch MR for the M856 and L110 Tracers.

Initial accuracy testing during Phase I showed Kart barrel number 014 to give slightly smaller round-to-round dispersion than did barrel 018; hence, barrel 014 was used for all subsequent BRL testing. The Phase II tests were conducted at four Mach numbers; 2.75, 1.9, 1.1, and 0.7, which roughly correspond to ranges of zero (muzzle), 300 metres, 700 metres, and 1000 metres, respectively, for the Ball ammunition. Ten data rounds of each type were fired, for a total forty round test program; of these, thirty-six rounds yielded useful aerodynamic and flight performance data. A half-muzzle type yaw inducer was used on some of the Mach 1.1 and Mach 0.7 firings, in an attempt to determine any significant aerodynamic non-linearities at transonic and subsonic speeds. All Phase II aeroballistic tests were fired in the BRL Aerodynamics Range, using the same Phase I weapon mounting shown in Figure 1.

The Phase III testing also used the weapon mounting system of Figure 1, but with the gun moved to one of the three firing positions at the BRL Transonic Range facility. The real-range limit-cycle yaw tests were conducted at ranges of 300, 600, and 800 metres; ten data rounds of each ammunition type were fired at the 300 and 600 metre ranges, and fifteen data rounds of each type were fired at the 800 metre range. Unfortunately, the tracer projectiles fogged the photographic film at 800 metres range, due to the long residence time of the subsonic bullets over the instrumentation; hence, no tracer limit-cycle data were obtained at 800 metres.

Figures 2 and 3 show sketches of the U.S. and FNB Ball bullets, and the U.S. and FNB tracers, respectively. The sketches reflect bullet contour measurements, made on the BRL Mann optical comparator. Table 1 lists the average measured physical characteristics of the four projectile types.

Selected prints of spark shadowgraphs of the four bullet types, from the limit-cycle yaw tests, are shown in Figures 4 through 8. Figure 4 shows the flowfield around the M855 and SS-109 projectiles at 300 metres range ($M_{\infty} \approx 1.9$), figure 5 shows similar shadowgraphs for the same projectiles at 600 metres ($M_{\infty} \approx 1.2$) and figure 6 shows 800 metre results ($M_{\infty} \approx 0.9$). Figure 7 is a comparison of flowfields at 300 metres range ($M_{\infty} \approx 1.9$) for the M856 and L110 tracers, and figure 8 shows a 600 metre shadowgraph ($M_{\infty} \approx 1.4$) for the L110 tracer projectile. No 600 metre data were obtained for the M856 tracer, due to the extremely large ammunition dispersion at that range.

III. ACCURACY FIRING TEST RESULTS

The ammunition provided to the BRL for testing consisted of Ball, SS-109, Lot FNB-83A-001-001, and Tracer, L110, Lot FNB-83A-001-001, for the Belgian-produced rounds; and Ball, XM855E1, Lot LC-83E300-5230, plus Tracer, XM856E1, Lot LC-83E300-5231, for the U.S. production ammunition. In addition to the ammunition provided by the SAW Office, the BRL procured, through open commercial sources, a sufficient quantity of Federal .223 Remington unprimed cases (LOT 8A-2132), CCI #450 small rifle primers, Sierra, .224", 52 grain Benchrest bullets, and IMR 3031 propellant (LOT 232AA09A), for use in checking the accuracy of the Kart test barrels.

The accuracy firing test results are summarized in Table 2. All groups fired were ten-shot groups, with one exception, as noted. The accuracy firings showed that Kart barrel No. 014 was slightly superior to No. 018, based on the performance with 52 grain Sierra Hollow Point Boattail (HPBT) bullets, although both barrels showed very small dispersion with the Sierra HPBT bullets. The FNB manufactured SS-109 Ball and L110 Tracer ammunition also easily met accuracy requirements, from either Kart barrel tested.

The first test fired with LCAAP produced M855 Ball ammunition, from Kart barrel No. 014, failed to meet specifications. The only group fired with LCAAP manufactured M856 Tracer ammunition did meet the accuracy requirement. Additional testing was performed, in which the M855 bullets were pulled, then loaded in commercial Federal cases, first with IMR 3031 propellant, then with the standard charge of LCAAP propellant. Both groups met the accuracy requirement. Finally, the Sierra HPBT bullets were loaded in LCAAP primed cases, with IMR 3031 propellant, and the results duplicated the commercial case results, which suggested that the LCAAP cartridge case was not a significant contributor to the accuracy problem.

Although the BRL accuracy firing test results are based on small sample sizes, the indications are that the act of pulling the M855 bullet and reseating it into the cartridge case, using straight-line seating dies, improved its performance markedly. Neither the cartridge case nor the propellant used appeared to have any significant effect on dispersion.

The conclusions reached from the accuracy test firings were (1) The Kart-manufactured Mann test barrels were not a significant contributor to the observed M855/M856 dispersion problem, (2) The lot of M855 Ball ammunition tested failed to meet accuracy specifications, (3) The bullet seating operation in LCAAP ammunition assembly was at least part of the cause of the observed M855 dispersion.

IV. AEROBALLISTIC TEST RESULTS

The free-flight spark photography range data were fitted to solutions of the linearized equations of motion and these results used to infer linearized

aerodynamic coefficients, using the methods of Reference 2. The actual projectile aerodynamic force-moment system often is not strictly linear. Given sufficient data the actual non-linear behavior can be determined from the range results.³ For the four 5.56mm NATO projectiles, sufficient data were obtained to permit determination of the effect of yaw level on the drag coefficient; no statistically significant values of the non-linear terms could be found for any of the transverse aerodynamic force or moment coefficients.

The round-by-round aerodynamic data obtained are listed in Tables 3 and 4, and the measured flight motion parameters are given in Tables 5 and 6. Three rounds of SS-109 ammunition previously fired in an Obermeyer Mann barrel are also included in Tables 3 and 5.

A. Drag Coefficient

The drag coefficient, C_D , is determined by fitting the time-distance measurements from the range flight. C_D is distinctly non-linear with yaw level, and the value determined from an individual flight reflects both the zero-yaw drag coefficient, C_{D_0} , and the induced drag due to the average yaw level of the flight. The drag coefficient variation is expressed as an even power series in yaw amplitude:

$$C_D = C_{D_0} + C_{D_{\delta^2}} \delta^2 + \dots$$

where C_{D_0} is the zero-yaw drag coefficient, $C_{D_{\delta^2}}$ is the quadratic yaw-drag coefficient, and δ^2 is the total angle of attack squared.

Analysis of the SS-109 and M855 drag coefficient data showed the two rounds to have essentially equal yaw-drag characteristics, but significantly different zero-yaw drag levels. Values of $C_{D_{\delta^2}} = 7.0$ at supersonic speeds, and $C_{D_{\delta^2}} = 9.8$ at subsonic speeds were used to correct the range data to zero-yaw conditions, for both the SS-109 and the M855. Figure 9 shows the variation of the zero-yaw drag coefficient, C_{D_0} , with Mach number for the two projectiles. The M855 design has about 8% more drag than the SS-109 at supersonic speeds, and this difference increases to approximately 20% at subsonic speeds. Since the M855 and SS-109 projectiles do not reach subsonic speeds except at ranges beyond 800 metres, the effect of the subsonic drag difference between the two designs would be observed only at extremely long ranges.

2. C. H. Murphy, "Data Reduction for the Free Flight Spark Ranges," Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, BRL Report No. 900, February 1954. (AD 35833)
3. C. H. Murphy, "The Measurement of Non-Linear Forces and Moments by Means of Free Flight Tests," Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, BRL Report No. 974, February 1956. (AD 93521)

Figure 10 is a similar plot of the zero-yaw drag coefficients for the M856 and L110 tracer projectiles. A least squares fit of the range data yielded the values $C_{D_{\delta^2}} = 5.7$ at supersonic speeds, and $C_{D_{\delta^2}} = 6.4$ at

subsonic speeds, for both tracer projectiles. Note that all tracers were essentially non-burning over the 50 metre flight observed in the BRL Aerodynamics Range, thus Figure 10 represents the drag coefficient behavior of non-burning tracers. Figure 10 also shows that the M856 and the L110 tracers have essentially identical zero-yaw drag, although the larger observed round-to-round variation in tracer drag coefficient could mask a slight difference in average drag level. The larger round-to-round scatter in Figure 10 is primarily due to the ogival boattail shape of the tracer bullets; minor round-to-round variations in surface finish along an ogival boattail lead to a variable boundary layer separation point, which in turn leads to larger than usual round-to-round variations in base drag.

B. Overturning Moment Coefficient

The range measured overturning moment coefficient, $C_{M_{\alpha}}$, is plotted against Mach number in Figure 11, for the Ball projectiles, and Figure 12 for the Tracer ammunition. Figure 11 shows that the SS-109 projectile has an overturning moment coefficient approximately 5% higher than does the M856, at supersonic speeds. Figure 12 shows no significant difference in overturning moment coefficient between the M856 and the L110 tracers.

C. Gyroscopic Stability Factor

The launch gyroscopic stability factors (S_g) for the Ball and Tracer projectiles, fired from the 7-inch twist Kart barrel, are shown in Figures 13 and 14. Figure 13 indicates equivalent launch S_g for the M856 and the SS-109 projectiles, which shows that differences in the physical characteristics of the two designs essentially offset the overturning moment coefficient difference observed in Figure 11. Figure 14 shows that launch gyroscopic stability factors for the M856 and the L110 tracer designs are equivalent.

Note that the decreased launch S_g at lower launch velocities, shown in both Figures 13 and 14, will never be observed in field firings, since the 5.56mm NATO projectiles are never fired at reduced muzzle velocities. The gyroscopic stability factors shown at the highest velocities tested are representative of actual ammunition performance at ambient field conditions.

All four projectiles tested have sufficient gyroscopic stability to permit firing at extreme cold weather (high air density) conditions, with no significant degradation in performance.

D. Lift Force Coefficient

The range values of lift force coefficient, $C_{L_{\alpha}}$, are shown for the Ball and Tracer projectiles respectively, in Figures 15 and 16. Figure 15 shows equivalent values of $C_{L_{\alpha}}$ for the SS-109 and the M856 projectiles, and Figure

16 shows no significant difference in lift force coefficient between the M856 and the L110 tracer designs.

E. Magnus Moment Coefficient

The Magnus moment coefficient, $C_{M_{p\alpha}}$, is plotted against Mach number in Figures 17 and 18, for the Ball and Tracer projectiles, respectively, and for small angles of attack. The Magnus moment coefficient for the M855 and SS-109 projectiles is a small positive quantity at supersonic speeds, and a substantial negative quantity at transonic speeds. The Magnus moment coefficient for the tracer projectiles is positive at supersonic speeds. Unfortunately, the epicyclic damping rates were poorly determined for the tracer designs at transonic and subsonic speeds; hence, no Magnus moment or pitch damping moment data were obtained for the tracers at lower velocities.

F. Pitch Damping Moment Coefficient

The pitch damping moment coefficient sum, $(C_{M_q} + C_{M_{\dot{\alpha}}})$ is plotted against Mach number in Figures 19 and 20, for the Ball and Tracer projectiles, respectively, and for small angles of attack. The pitch damping moment coefficient sum for the M855 and SS-109 projectiles is substantially negative at supersonic speeds, and tends toward zero at transonic speed. The pitch damping moment for the tracer projectiles is a relatively large negative quantity at supersonic speeds. Since a negative value of $(C_{M_q} + C_{M_{\dot{\alpha}}})$ causes damping of the yawing motion, Figures 19 and 20 show generally favorable pitch damping properties of all the 5.56mm NATO projectiles at supersonic speeds.

G. Damping Rates

The damping rates, λ_F and λ_S , of the fast and slow yaw modes indicate the dynamic stability of a projectile. Negative λ 's indicate damping; a positive λ means that its associated modal arm will grow with increasing time.

Figures 21 and 22 show the fast and slow arm damping rates for the M855 and SS-109 Ball projectiles, at small angles of attack. Figure 21 shows that the fast arm is damped at all velocities tested. Figure 22 shows the slow arm to be damped at high supersonic speeds, but tending toward a weak undamping at low supersonic and transonic speeds. Thus, Figure 22 suggests the possibility of a slow-mode limit cycle yaw at low supersonic and transonic speeds, for both the M855 and SS-109 projectiles.

Figures 23 and 24 show the fast and slow arm damping rates for the M856 and L110 Tracer projectiles, at small angles of attack. Both yaw modes appear to be strongly damped at supersonic speeds. Unfortunately, the damping rates of the tracer projectiles fired at transonic and subsonic speeds were poorly determined, thus no data were obtained for the lower velocity regions.

V. LIMIT-CYCLE YAW TEST RESULTS

The final phase of the BRL tests of 5.56mm NATO ammunition was the real-range, limit-cycle yaw and striking velocity determination. The limit-cycle yaw testing was conducted during March 1984, at the BRL Transonic Range facility, using three of the Aerodynamics Range spark photography stations. Figure 25 is a photograph of the experimental facility used for limit-cycle yaw testing. The direction of bullet travel in Figure 25 is from upper left, through the barricades and spark photography stations, and into the bullet trap at lower right.

If a projectile successfully negotiates the instrumentation shown in Figure 25, and all stations are triggered, three measurements of striking yaw and two determinations of striking velocity are obtained. Occasionally, two of the three spark stations will trigger, and two determinations of yaw and one velocity are obtained. No data were obtained for the L110 tracer at 800 metres range, since the long residence time of the bright tracer over the photographic plates caused excessive fogging of the film. No data were obtained for the M856 tracer at either 600 metres or 800 metres range, due to the large round-to-round dispersion of the M856 ammunition. A tabulated summary of all the data obtained from the limit-cycle yaw testing is given in Table 7.

The striking velocity and limit-cycle yaw behavior of the 5.56mm NATO ammunition is shown in Figures 26 through 29. Figures 26 and 27 show striking velocity and striking yaw for the SS-109 and M855 projectiles, respectively, from the Kart Mann barrel, No. 014. The vertical bars shown on the striking yaw plots represent limits of plus and minus one standard deviation. Figure 28 is a similar plot of striking velocity and striking yaw for the two tracer projectiles, also from the Kart Mann barrel. The 856 tracer projectile shows approximately 6 metres/second higher striking velocity than does the L110, out to 300 metres range. Note that the striking yaw histories of the L110 and M856 projectiles appear to be essentially identical, out to 300 metres.

Figure 29 is a plot of striking velocity and striking yaw versus range, for the SS-109 projectile, fired from the XM249E1 Squad Automatic Weapon (SAW), in June 1981. The ammunition used was Lot 01 FNB 81, and the SS-109/XM249E1 test was conducted with funding provided by the U.S. Army Materiel Systems Analysis Activity (AMSAA). Note the excellent agreement in striking velocity history between the two weapons, as indicated in Figures 26 and 29. The limit-cycle yaw, at ranges beyond 300 metres, appears to be slightly greater from the Kart Mann barrel than that observed three years earlier from the XM249E1. It is not possible to infer from the present data if the difference in observed limit-cycle yaw behavior is due to weapon/rifling differences, or the lot-to-lot differences in the SS-109 ammunition.

VI. COMMENTS ON AMMUNITION DISPERSION

One of the principal problems encountered in U.S. production of the 5.56mm NATO ammunition has been excessively high rejection rates of LCAAP manufactured ammunitions lots. In 1982-83, several lots of M855 Ball ammunition failed to meet the accuracy specification. The M855 accuracy problem has since been corrected; however, current lots of LCAAP produced M856 Tracer ammunition are showing excessive rejection rates, again due to failure on

the accuracy specification. In nearly all cases, the FNB manufactured SS-109 Ball and L110 Tracer ammunition, fired simultaneously as control rounds, meet or exceed the U.S. accuracy specifications.

The largest contributing factors to dispersion in modern small arms ammunition are the lateral throwoff and the aerodynamic jump, produced by projectile static and dynamic unbalance, respectively. Of these two, the aerodynamic jump due to dynamic unbalance is generally the predominant effect, and we will examine some of the causes and consequences of dynamic unbalance in modern small arm projectiles.

If a bullet jacket varies in wall thickness around its circumference, and the jacket and core are of different density materials, an unbalance is introduced in proportion to the jacket wall eccentricity. If the lateral meridian plane containing the eccentricity is not held constant, both a static and a dynamic unbalance are introduced. In addition, if the bullet has a two-piece core, and the front and rear core sections are of widely different density materials, even a relatively small jacket wall eccentricity can introduce a large dynamic unbalance in the bullet.

In their classical text, Exterior Ballistics,⁴ McShane, Kelley, and Reno derive the aerodynamic jump effect, due to either an in-bore yaw, or an analogous dynamic unbalance in the projectile. The aerodynamic jump is the amount by which the direction of motion of the projectile is changed, and is given as an angle in radians. The result, from Chapter XII of Exterior Ballistics, is converted to the modern aeroballistic nomenclature:

$$\text{Jump} = \left(\frac{2\pi}{n} \right) (k_t^2 - k_a^2) \left(\frac{C_{L\alpha}}{C_{M\alpha}} \right) \epsilon$$

where Jump = magnitude of trajectory deflection (radians)
 n = twist of rifling (calibers/turn)

$$k_t^2 = I_y / m d^2$$

$$k_a^2 = I_x / m d^2$$

I_y = projectile transverse moment of inertia

I_x = projectile axial moment of inertia

m = projectile mass

d = projectile reference diameter

C_L = lift force coefficient

$C_{M\alpha}$ = overturning moment coefficient

ϵ = dynamic unbalance angle, or in-bore yaw due to bullet tilt
in cartridge case (radians)

4. E. J. McShane, J. L. Kelley, and F. V. Reno, Exterior Ballistics, University of Denver Press, 1953.

For a given dynamic unbalance angle, the quantity $\left[\left(\frac{2\pi}{n} \right) (k_t^2 - k_a^2) \left(\frac{C_{L\alpha}}{C_{M\alpha}} \right) \right]$

can be considered as a dispersion "sensitivity factor," and we will now evaluate this factor for several 5.56mm projectiles. The M193 Ball and M196 Tracer rounds, fired from the M16A1 rifle with 12-inch twist of rifling, will be compared with the M855 Ball and M856 Tracer projectiles fired from the 7-inch twist rate of the M16A2 or the M249. The results are given in Table 8.

Comparison of the dispersion sensitivity factors in the last column of Table 8 shows that the dynamic unbalance of the M855 bullet must be held to approximately 60% of that present in M193 projectiles, in order to equal the M193 dispersion. The same comparison of old and new tracer ammunition shows that the M856 dynamic unbalance cannot exceed 50% of that present in M196 tracers, if comparable dispersions are to be obtained. Since the dynamic unbalance in all the above bullets is approximately proportional to jacket wall eccentricity, it is apparent that tolerances in jacket wall thickness for the new 5.56mm NATO projectiles need to be held at half the levels permitted for the older M193/M196 family of 5.56mm ammunition. The aerodynamic jump due to bullet tilt in the cartridge case is analagous to that of dynamic unbalance; thus, bullet tilt in the seating operation of M855/M856 ammunition should be held to half that allowed for the M193/M196 family.

VII. CONCLUSIONS AND RECOMMENDATIONS

The Kart manufactured Mann test barrels supplied to LCAAP are not a significant contributor to the observed M855/M856 dispersion problem.

The bullet seating operation in LCAAP ammunition assembly is part of the cause of observed M855/M856 dispersion, and some effort needs to be made in the direction of reduced bullet tilt in assembled ammunition.

Tolerances in bullet jacket wall thickness for the M855/M856 need to be held to approximately half those permitted for the older M193/M196 projectiles, to insure satisfactory accuracy with the new 5.56mm NATO ammunition.

The zero-yaw drag coefficient of the M855 Ball projectile is approximately eight percent higher than that of the SS-109 design, at supersonic speeds. The drag coefficients of the M856 and L110 tracer projectiles appear to be essentially identical at supersonic speeds.

All four 5.56mm NATO projectiles tested have sufficient gyroscopic stability, when fired from the 7-inch twist of rifling, to permit firing at extreme cold weather (high air density) conditions, with no significant degradation in performance.

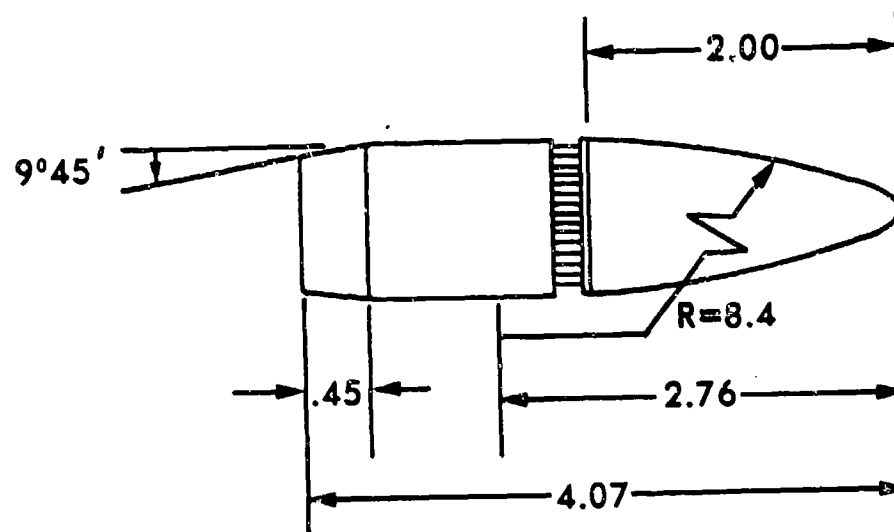
The M855/SS-109 Ball projectiles show good yaw damping properties for both the fast and slow yaw modes, at supersonic speeds. The slow yaw mode shows weak undamping at small yaw levels for both Ball projectile designs at transonic speeds.

The M855/SS-109 Ball projectiles show limit-cycle yaw levels growing from approximately 0.5 degree at 300 metres range, to approximately 6 degrees at 800 metres range. The tracer projectiles show yaw levels damping to approximately 0.5 degree at 300 metres range, and remaining at this level out to 600 metres range.

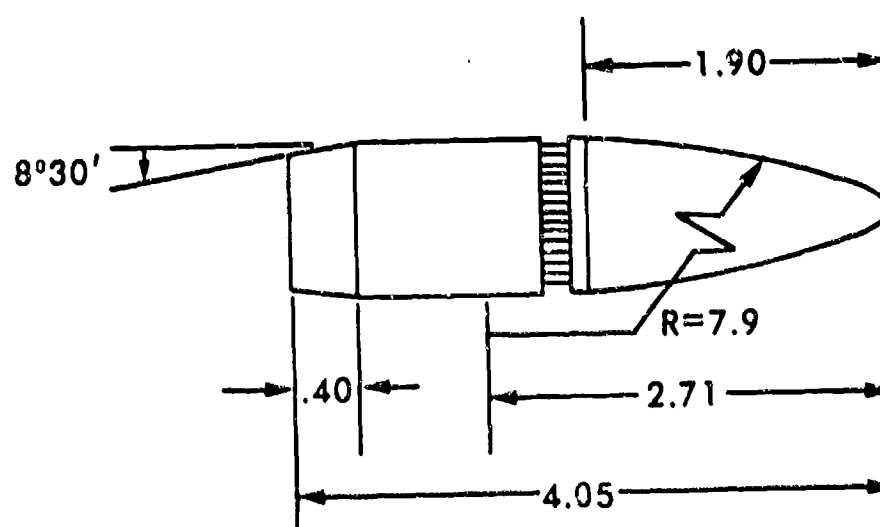


Figure 1. Photograph of the Mann Barrel and Frankford Rest

SS-109



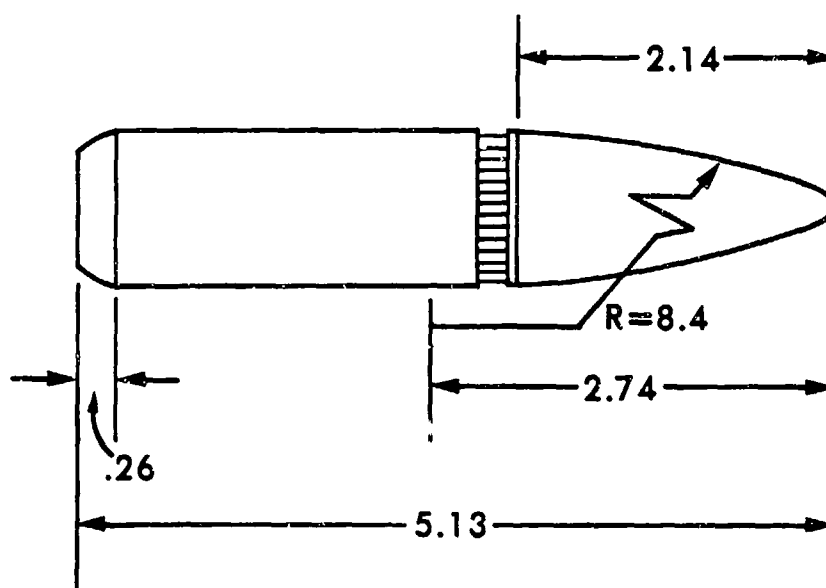
M855



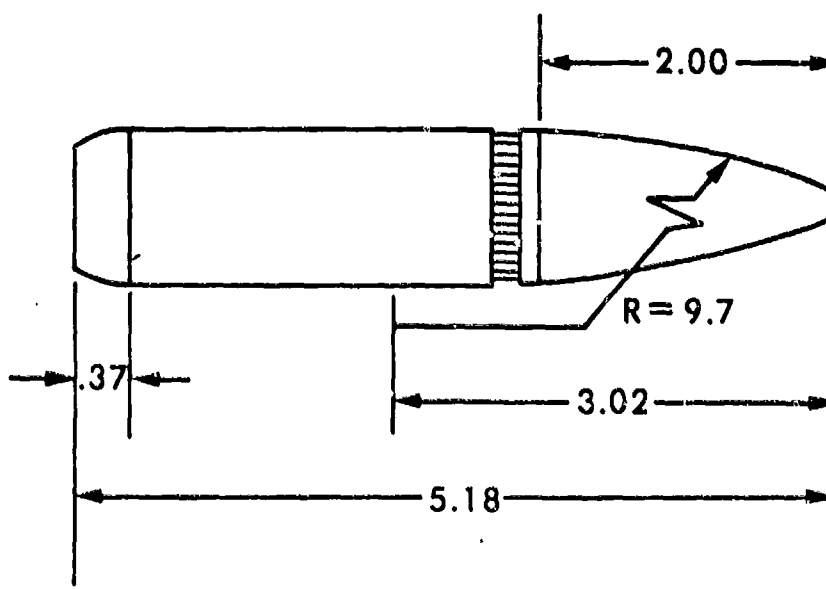
ALL DIMENSIONS IN CALIBERS
(1 CALIBER = 5.69 mm)

Figure 2. Sketch of SS-109 and M855 Ball Projectiles

L110



M856



ALL DIMENSIONS IN CALIBERS
(1 CALIBER = 5.69 mm)

Figure 3. Sketch of L110 and M856 Tracer Projectiles

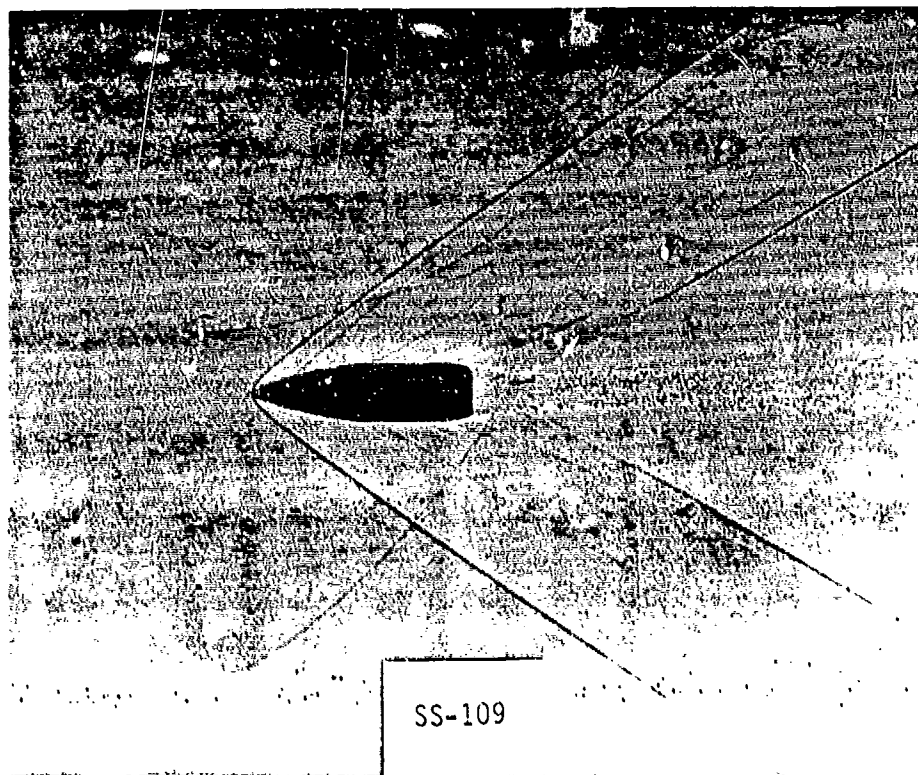
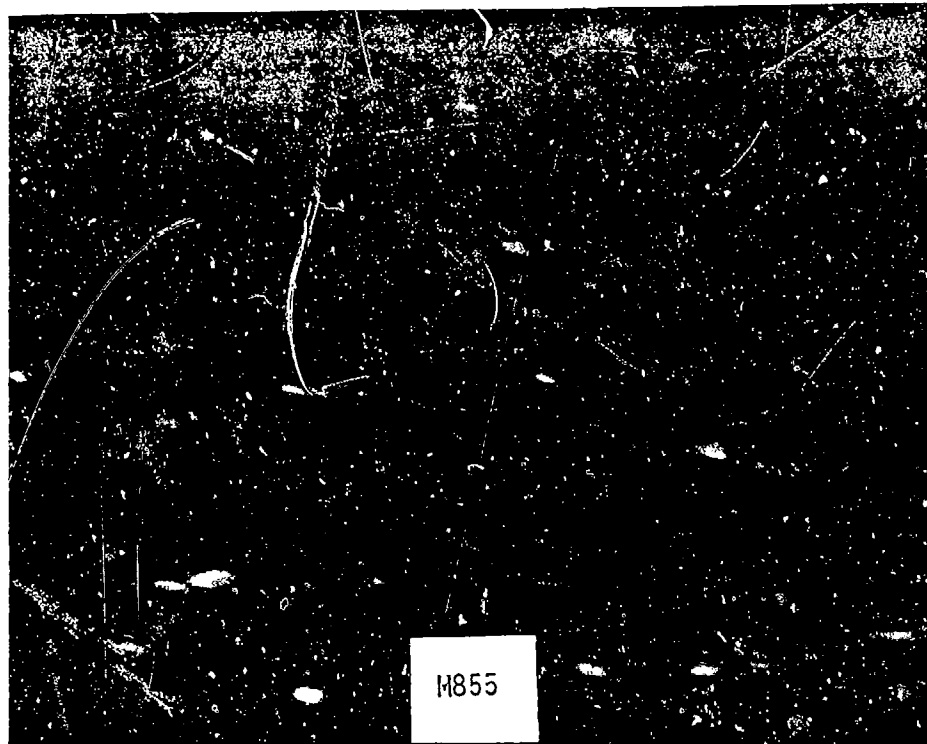


Figure 4. Shadowgraphs of M855 and SS-109 Projectiles at Mach 1.9

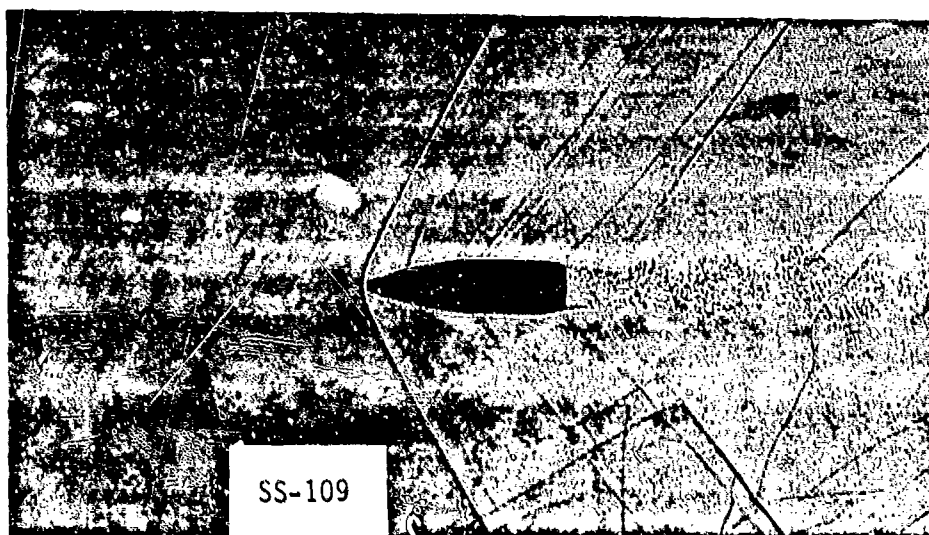
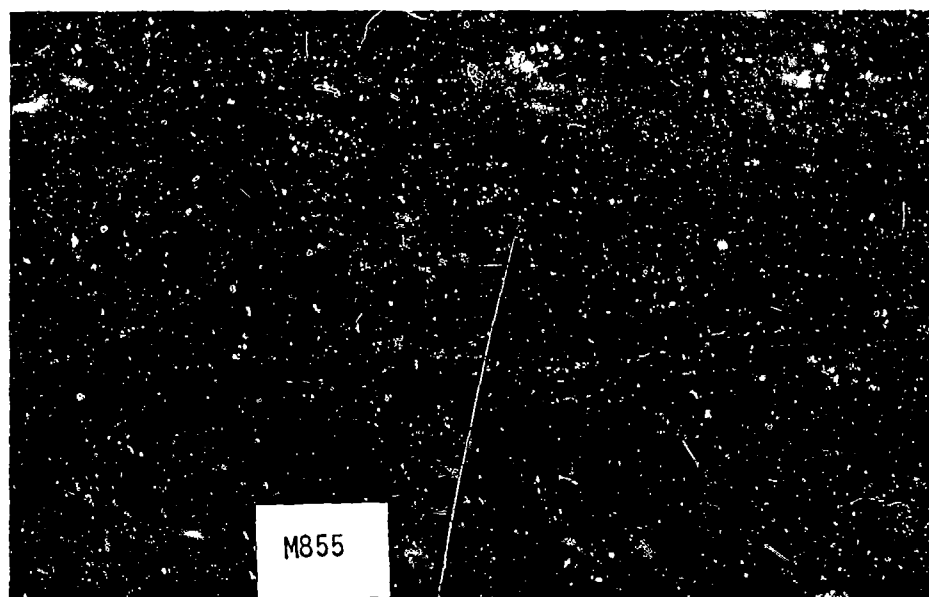


Figure 5. Shadowgraphs of M855 and SS-109 Projectiles at Mach 1.2

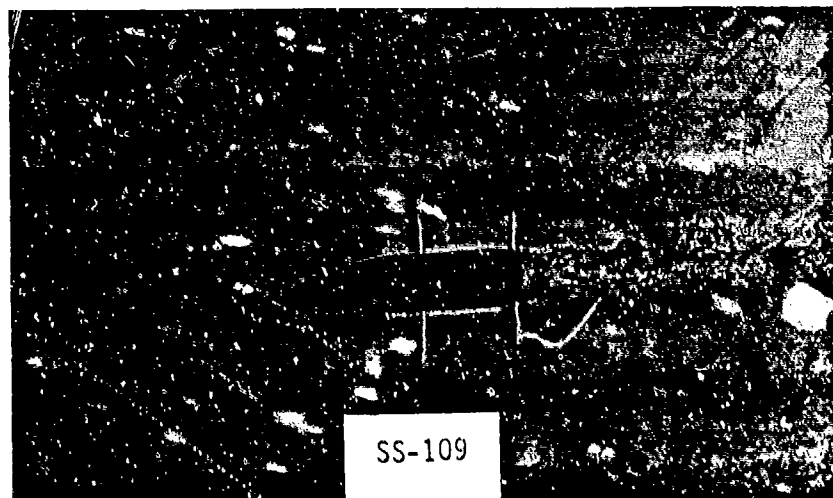
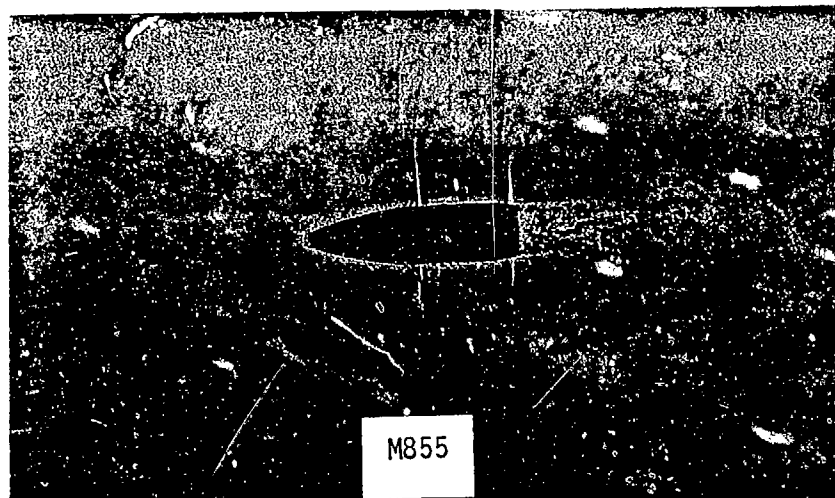


Figure 6. Shadowgraphs of M855 and SS-109 Projectiles at Mach 0.9

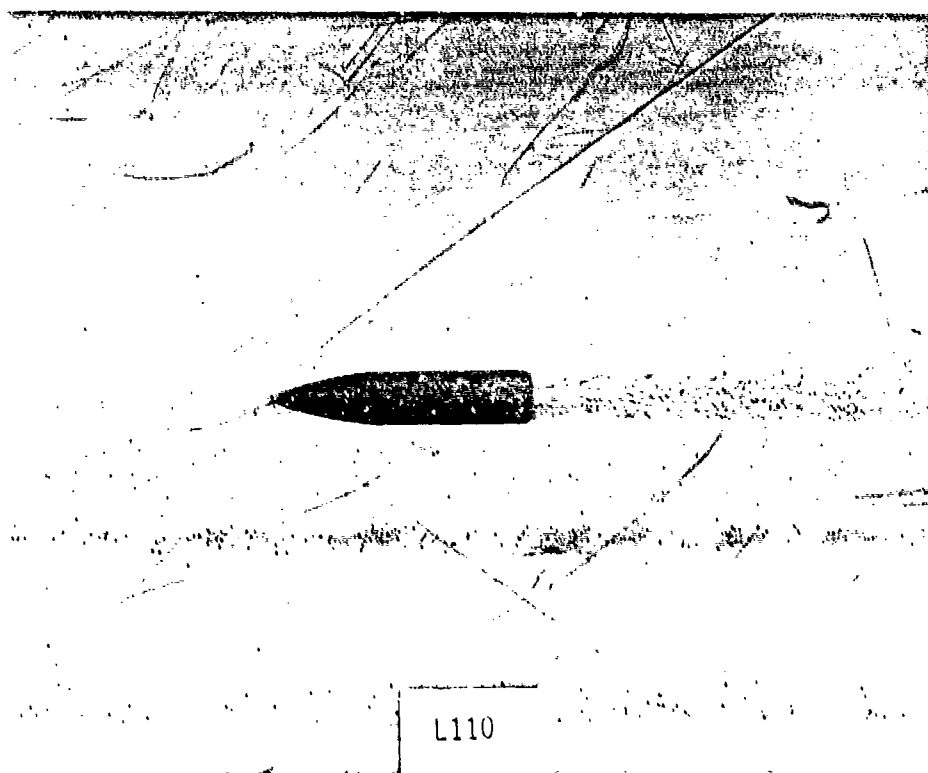
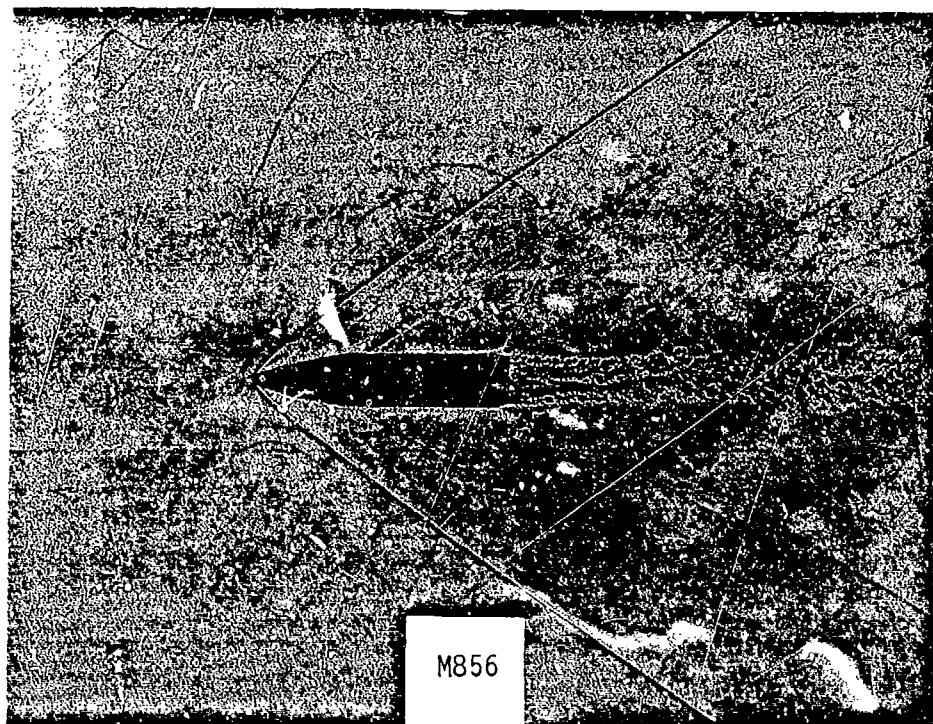


Figure 7. Shadowgraphs of M856 and L110 Projectiles at Mach 1.9

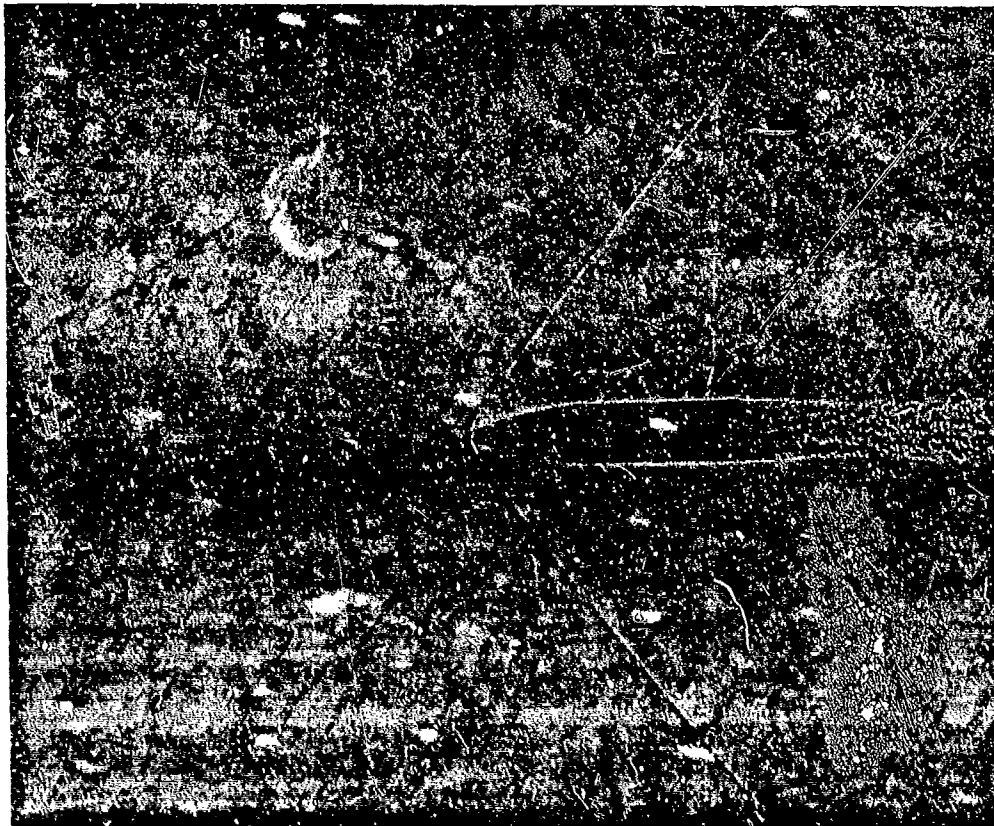


Figure 8. Shadowgraph of L110 Projectile at Mach 1.4

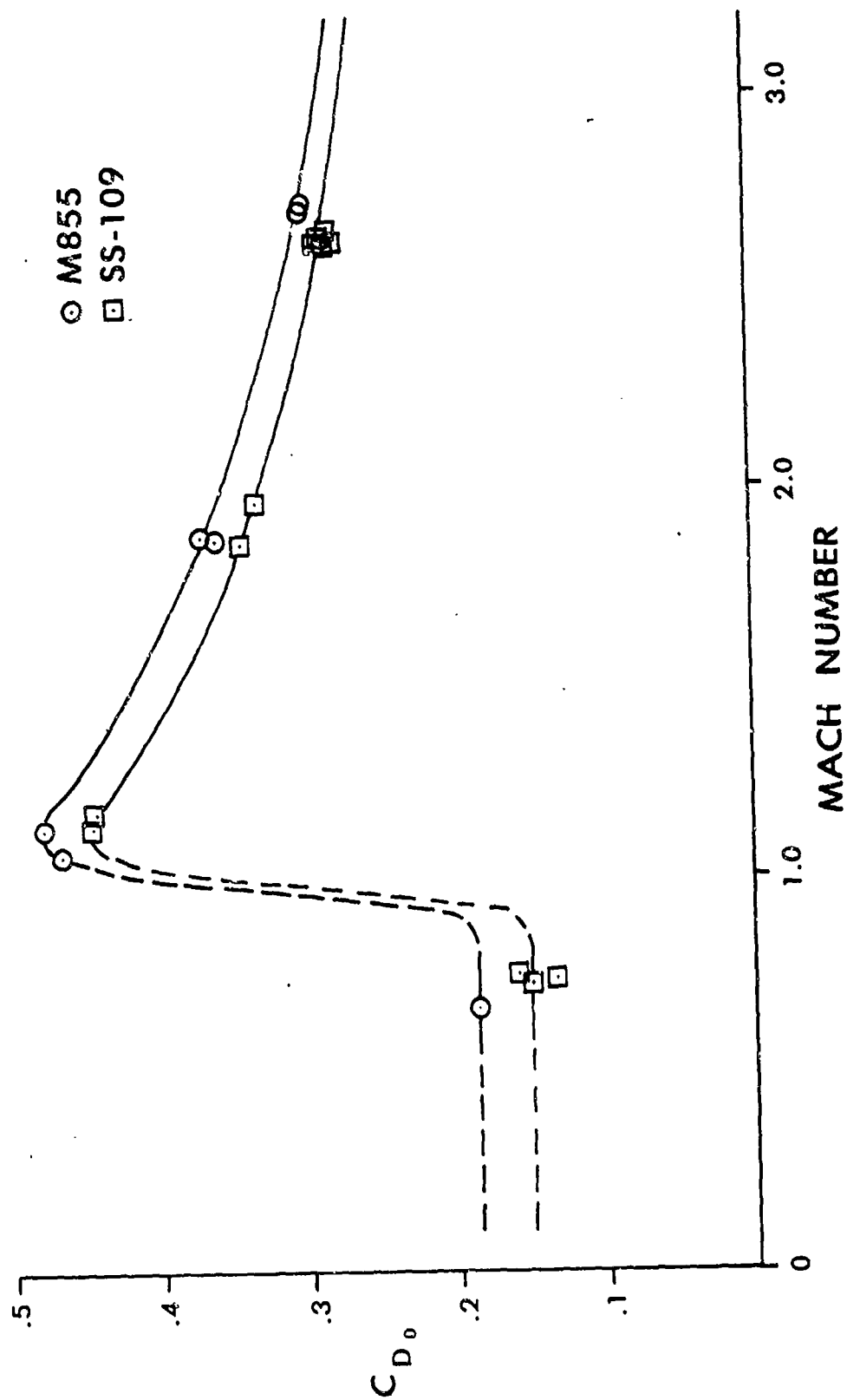


Figure 9. Zero-Yaw Drag Force Coefficient versus Mach Number, M855/SS-109

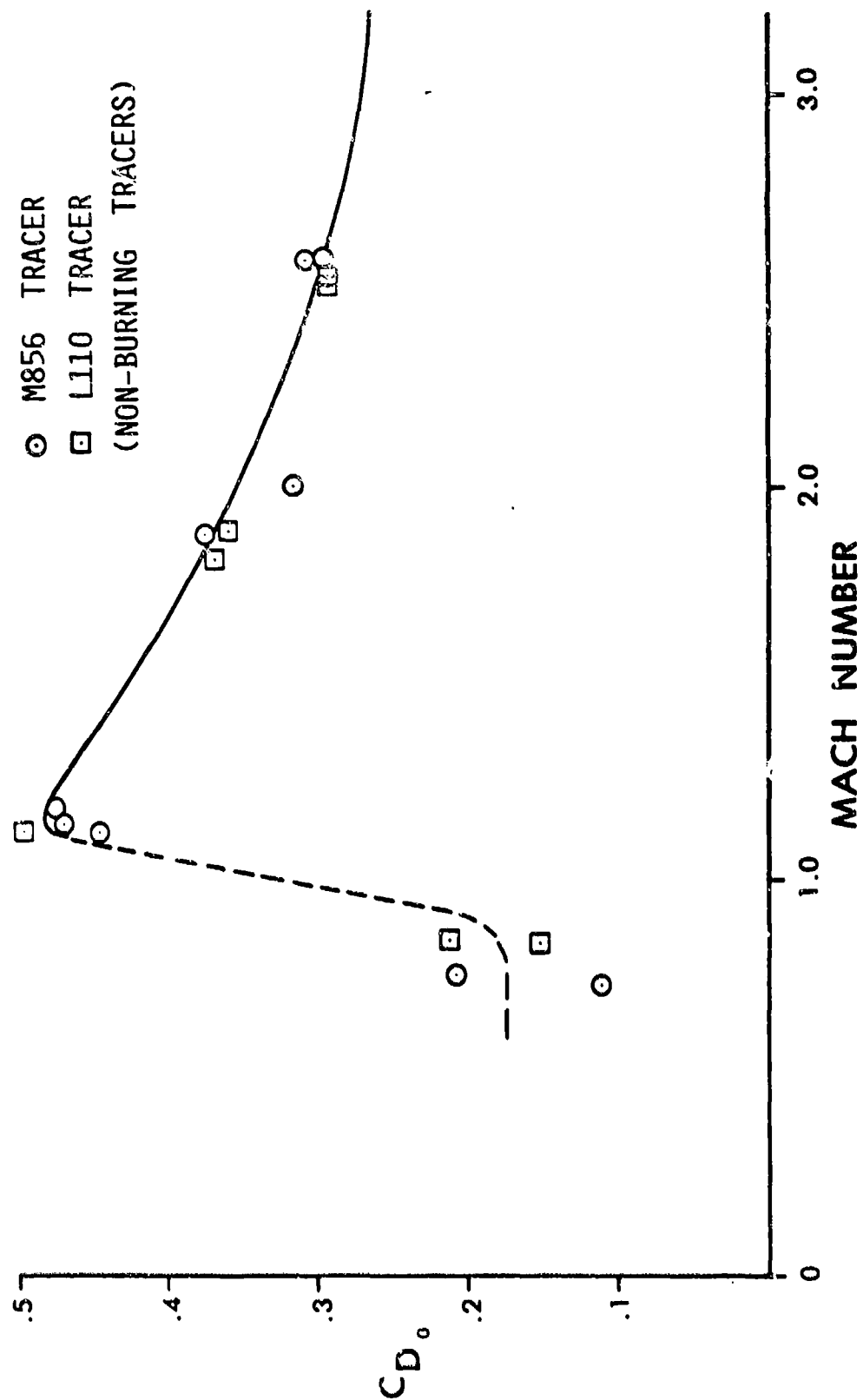


Figure 10. Zero-Yaw Drag Force Coefficient versus Mach Number, M856/L110

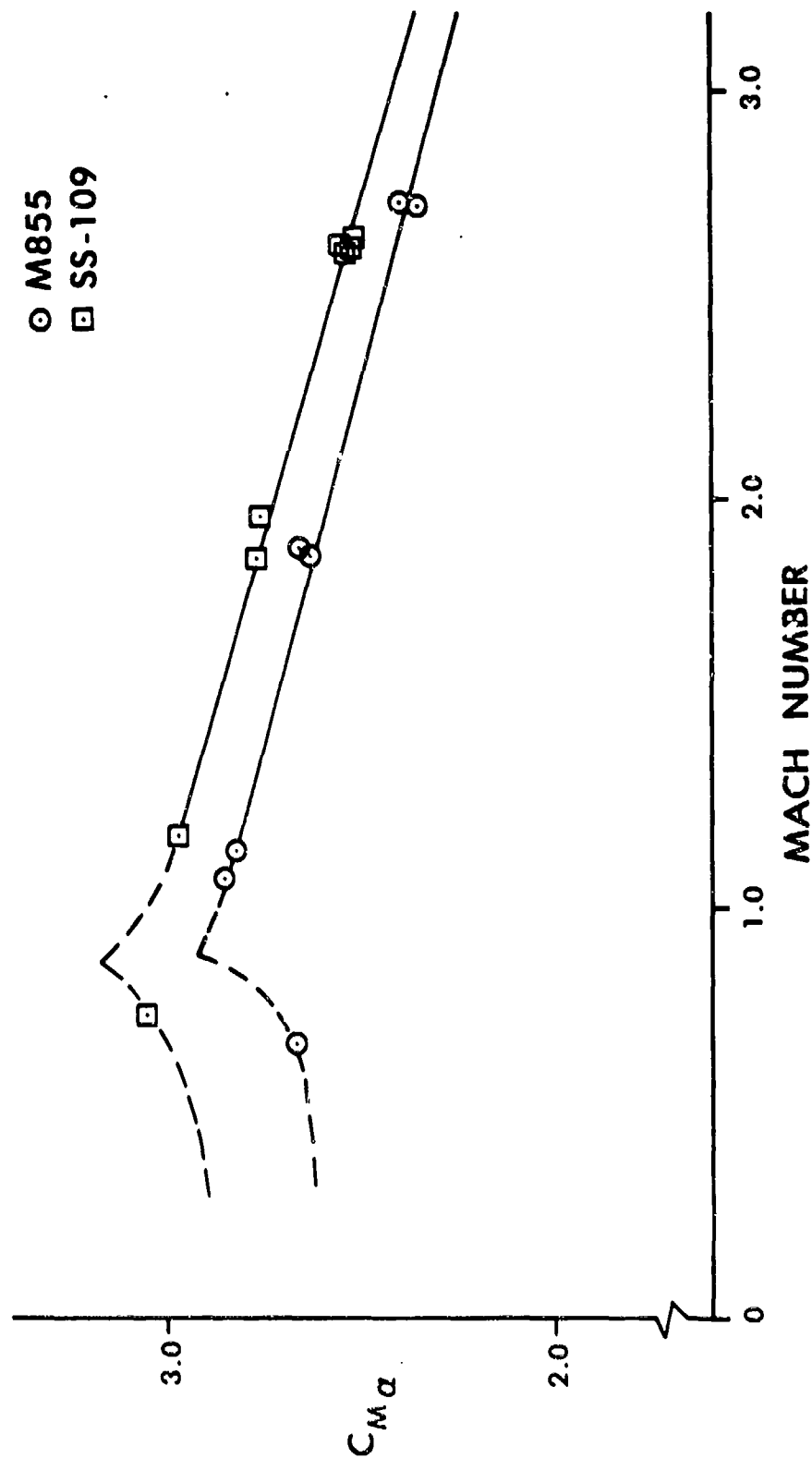


Figure 11. Overturning Moment Coefficient versus Mach Number, M855/SS-109

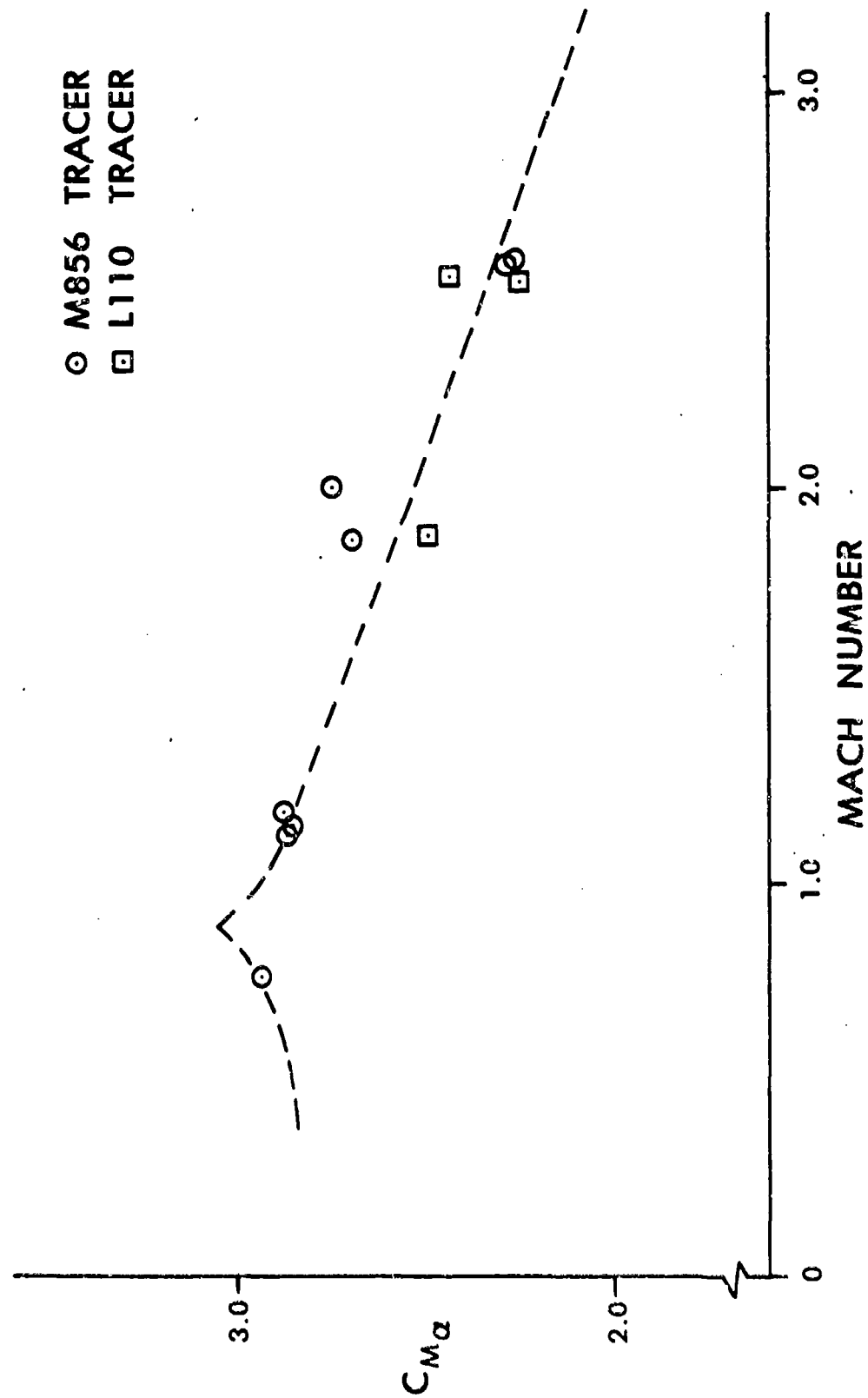


Figure 12. Overturning Moment Coefficient versus Mach Number, M856/L110

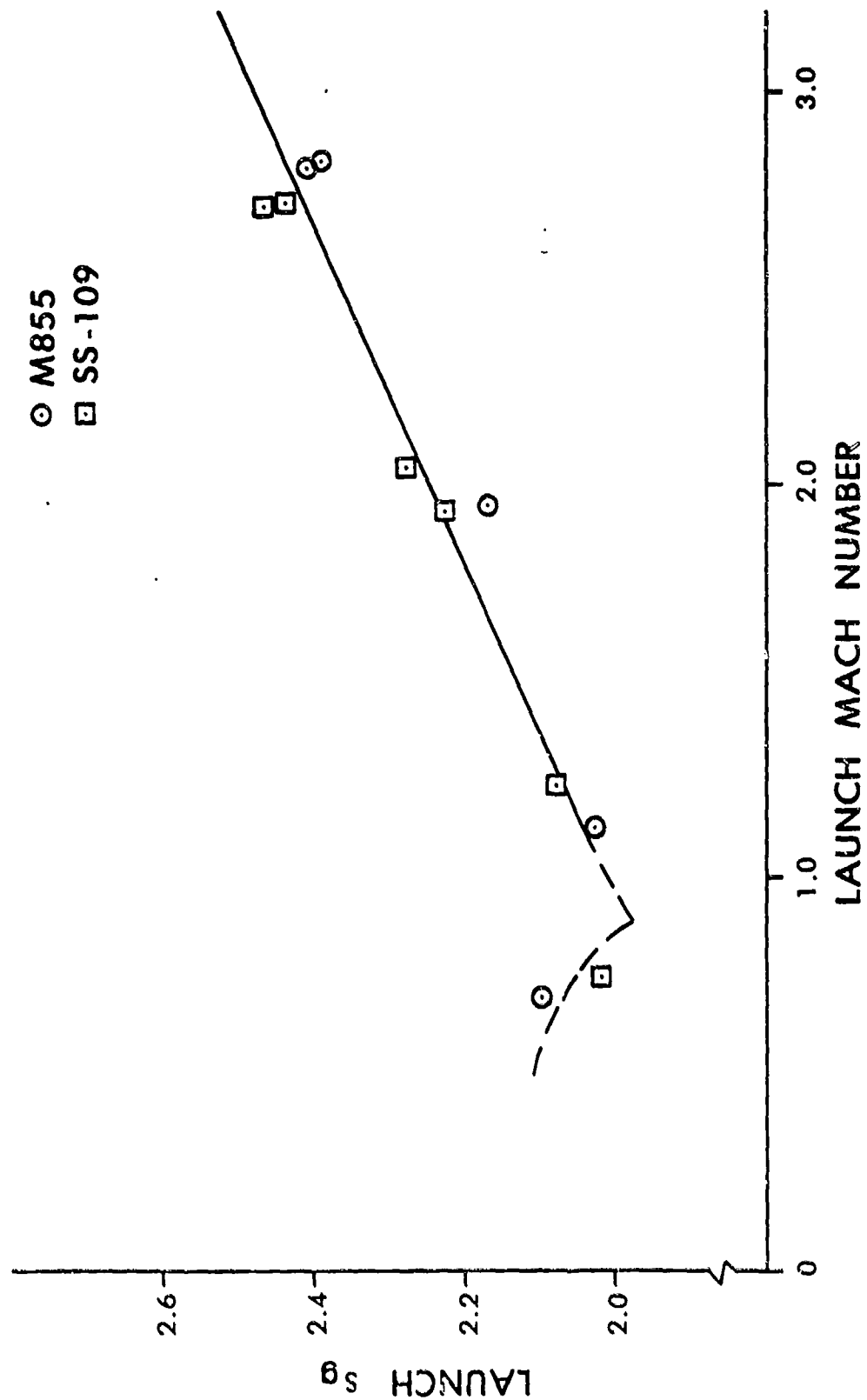


Figure 13. Gyroscopic Stability Factor versus Mach Number, M855/SS-109

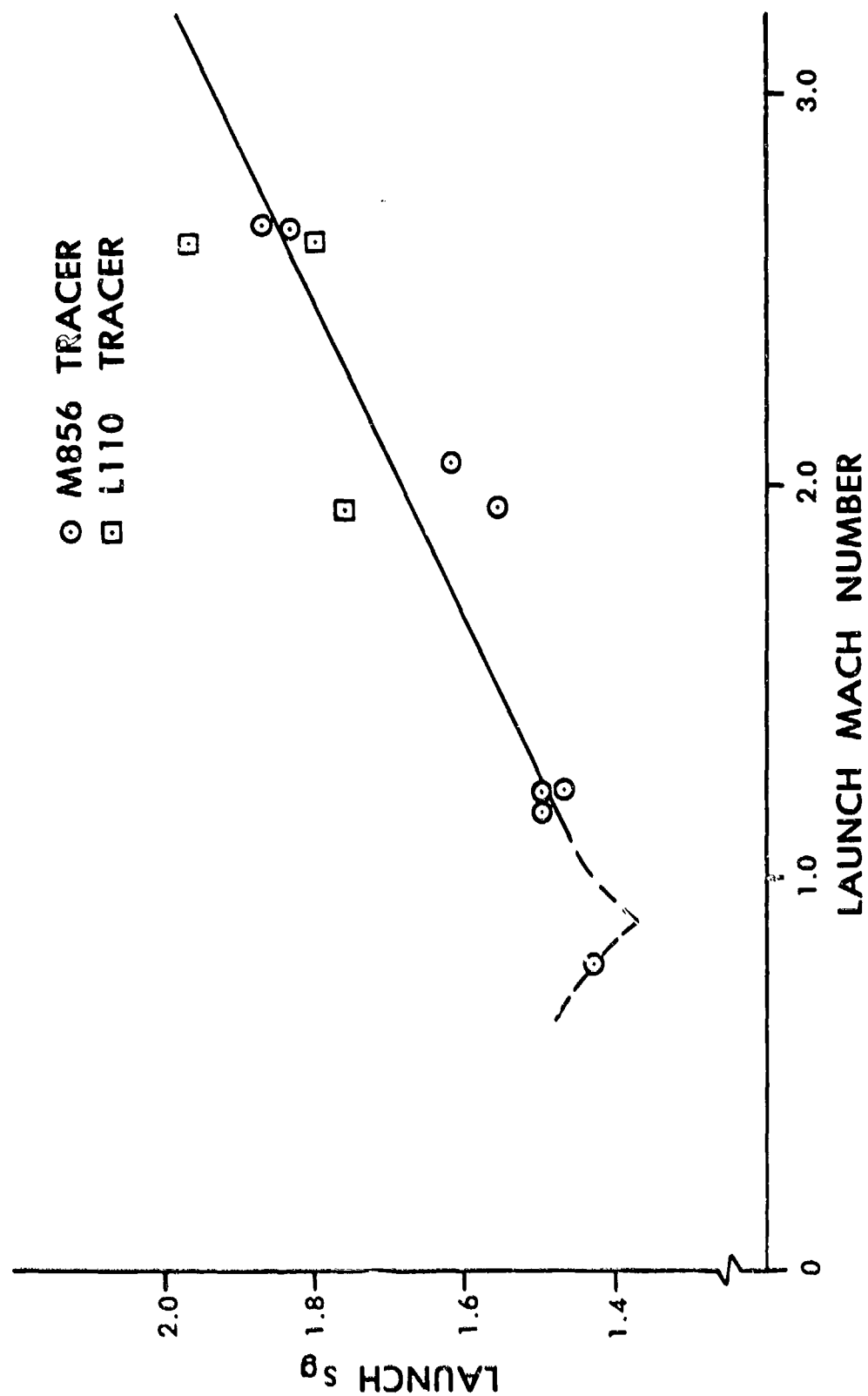


Figure 14. Gyroscopic Stability Factor versus Mach Number, M856/L110

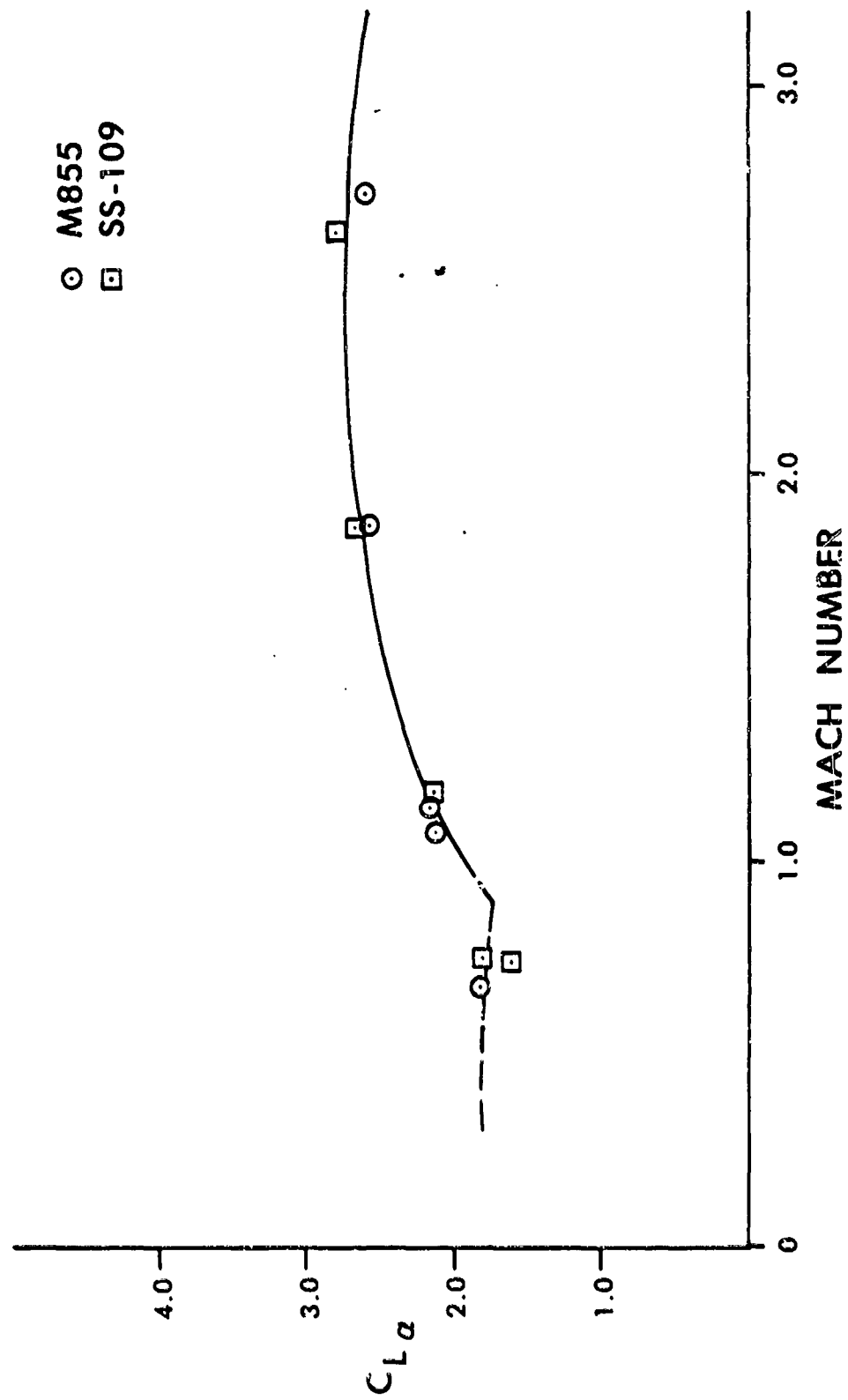


Figure 15. Lift Force Coefficient versus Mach Number, M855/SS-109

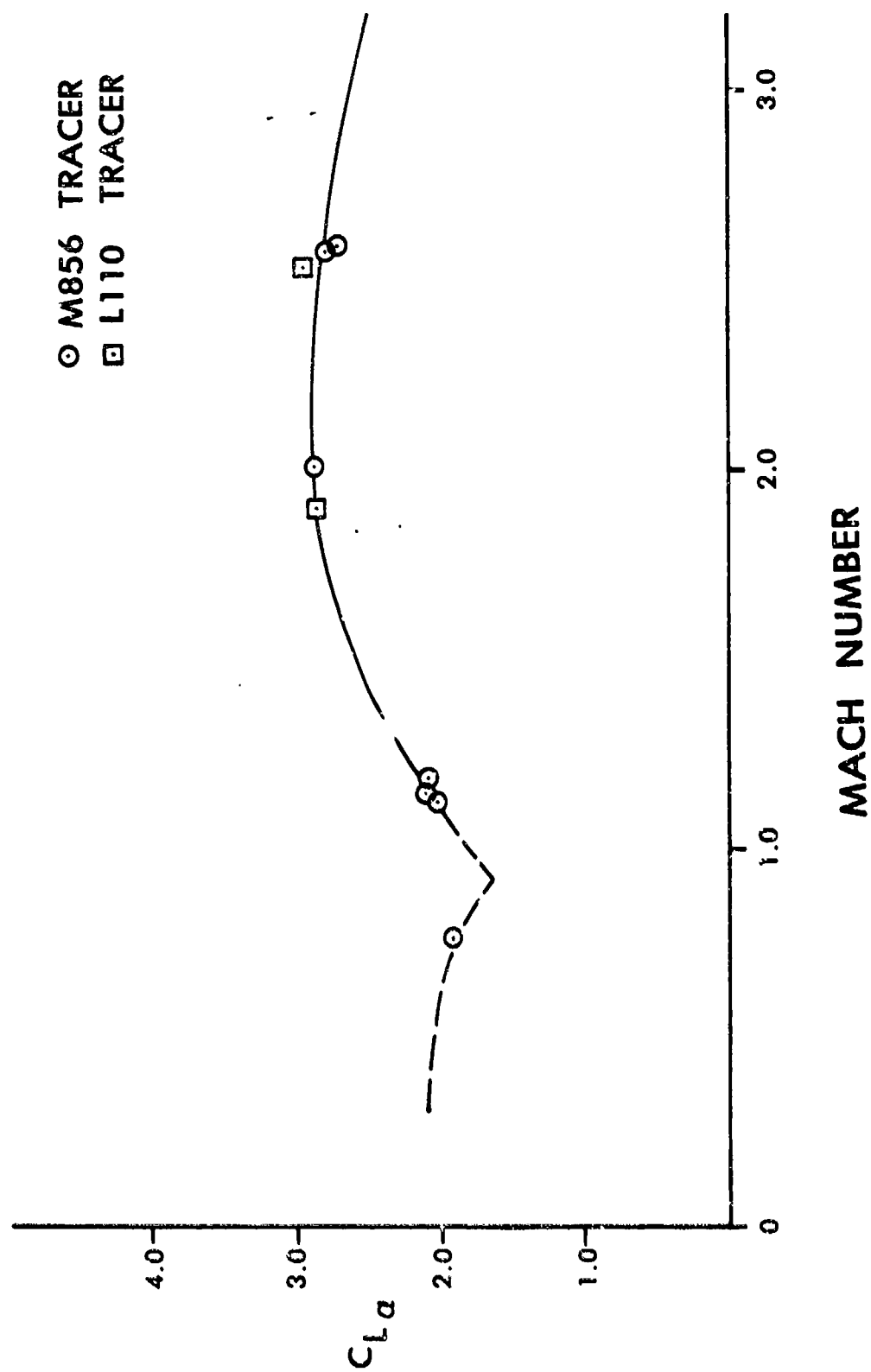


Figure 16. Lift Force Coefficient versus Mach Number, M856/L110

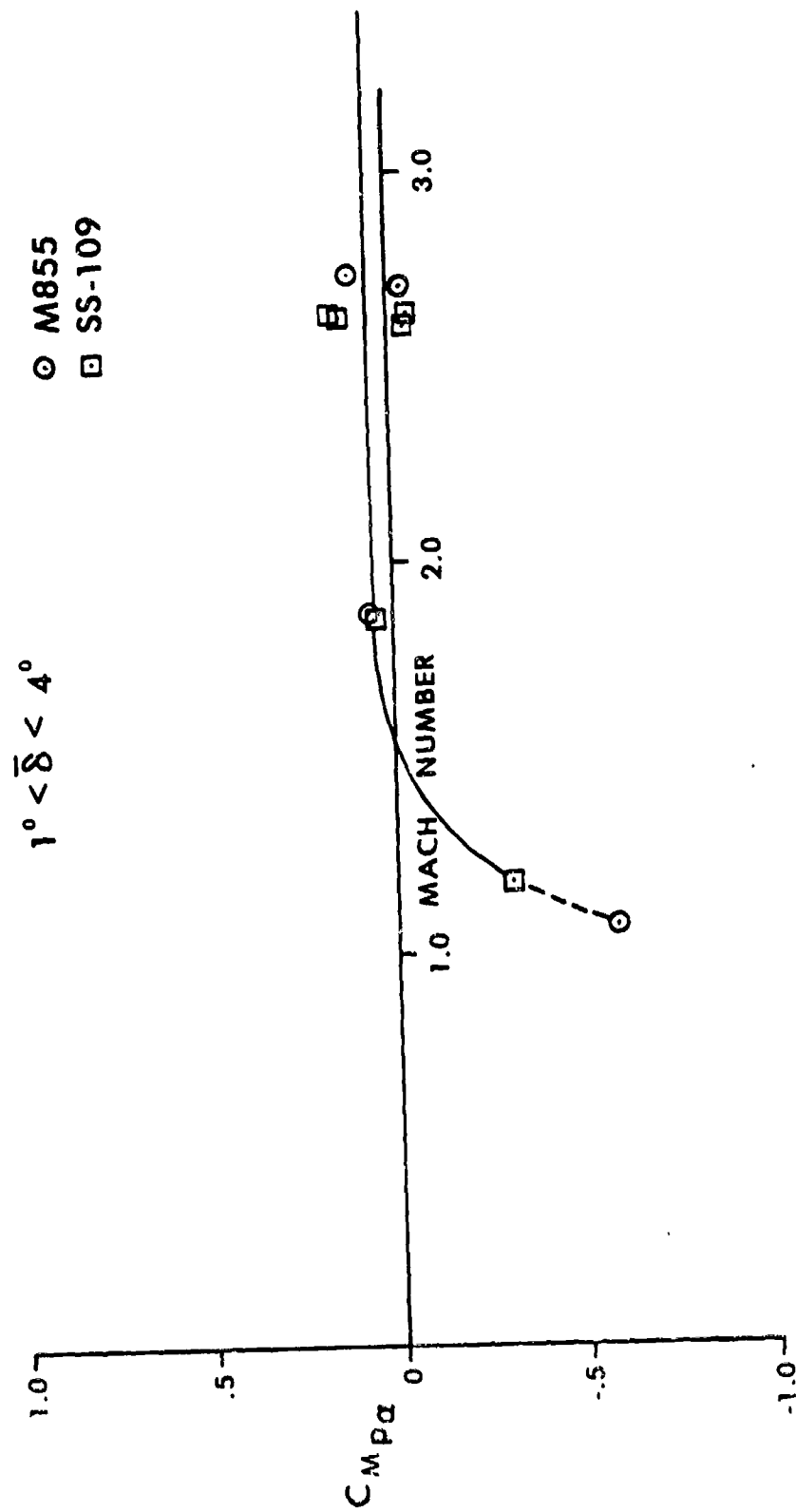


Figure 17. Magnus Moment Coefficient versus Mach Number,
M855/SS-109

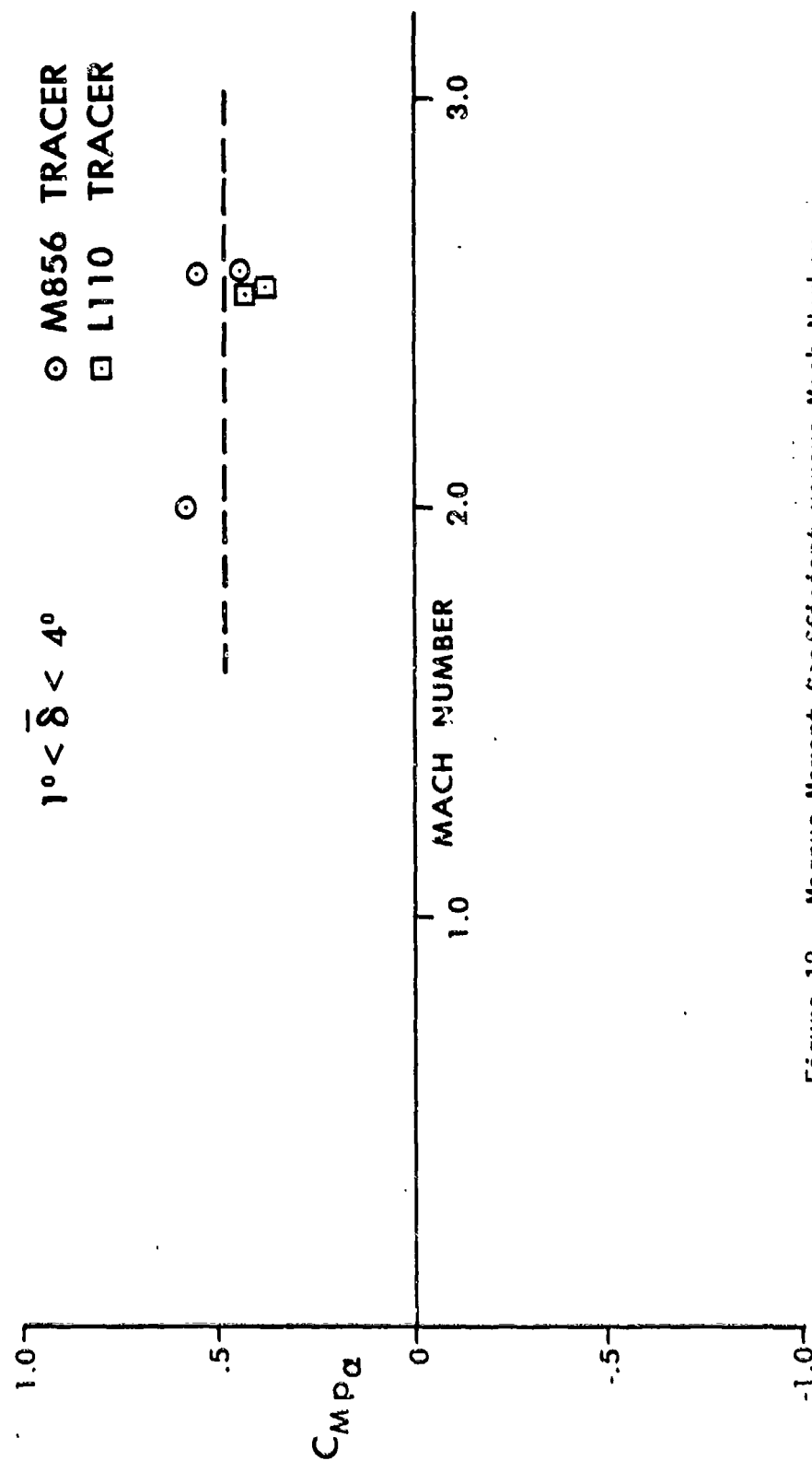


Figure 18. Magnus Moment Coefficient versus Mach Number,
M856/L110

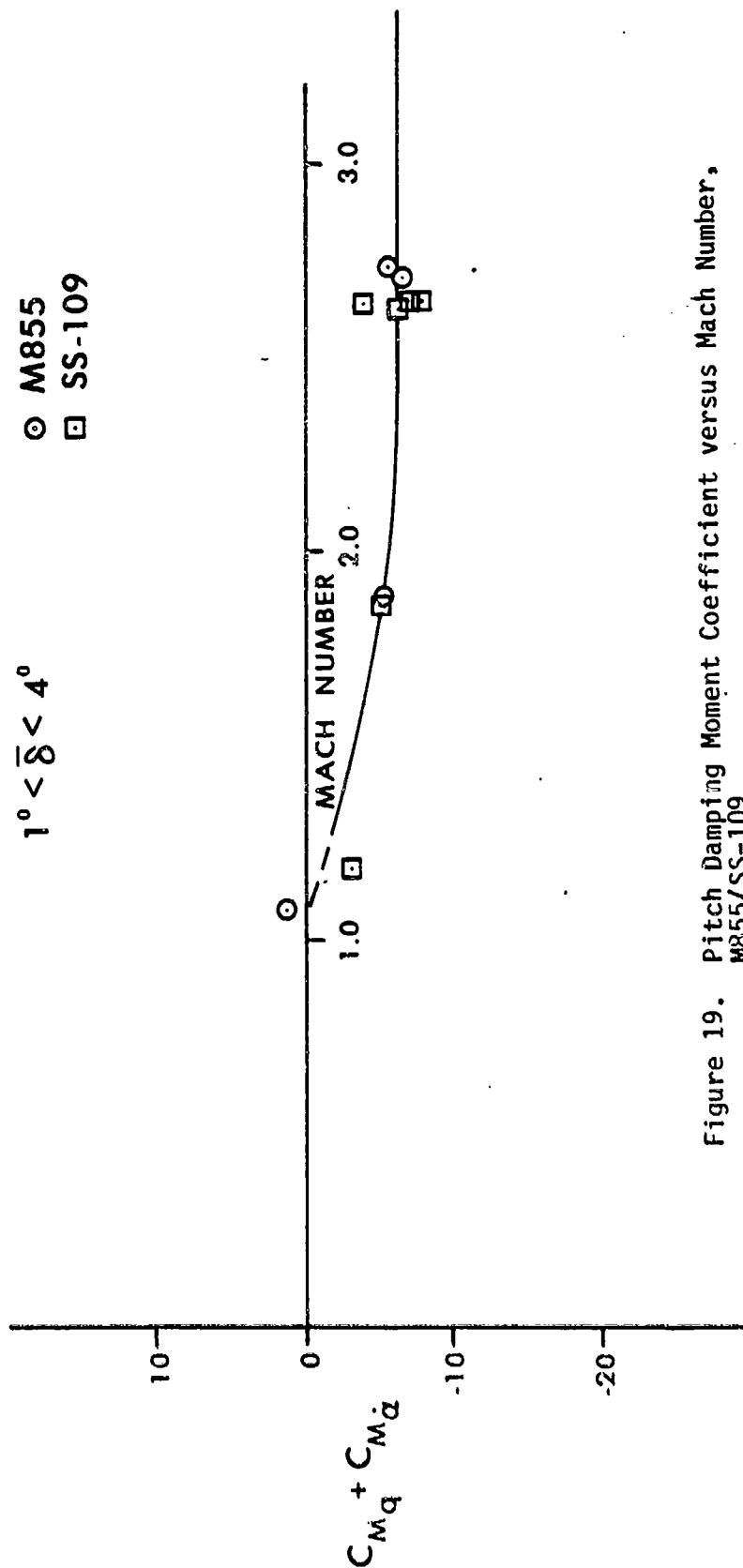


Figure 19. Pitch Damping Moment Coefficient versus Mach Number, M855/SS-109

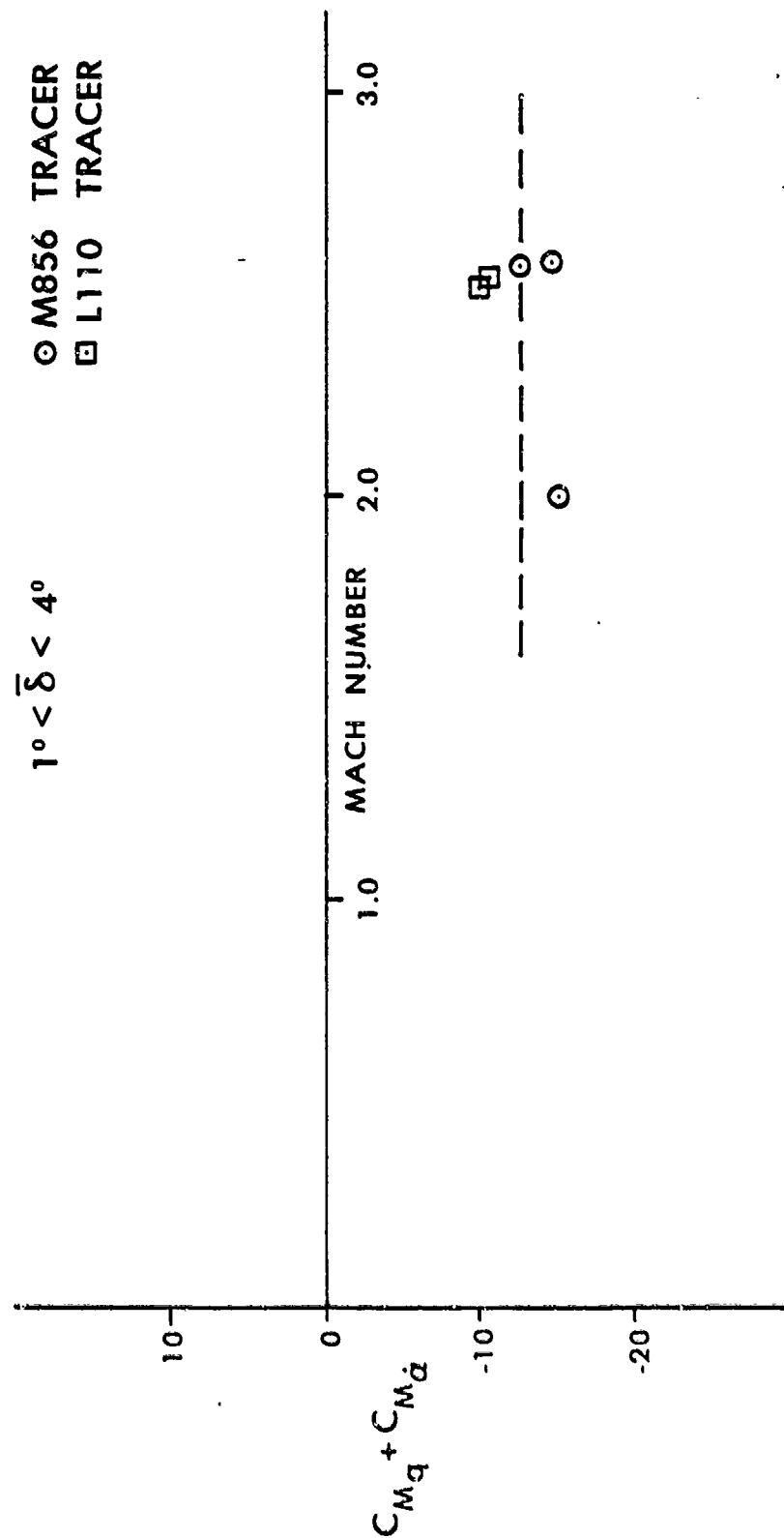


Figure 20. Pitch Damping Moment Coefficient versus Mach Number, M856/L110

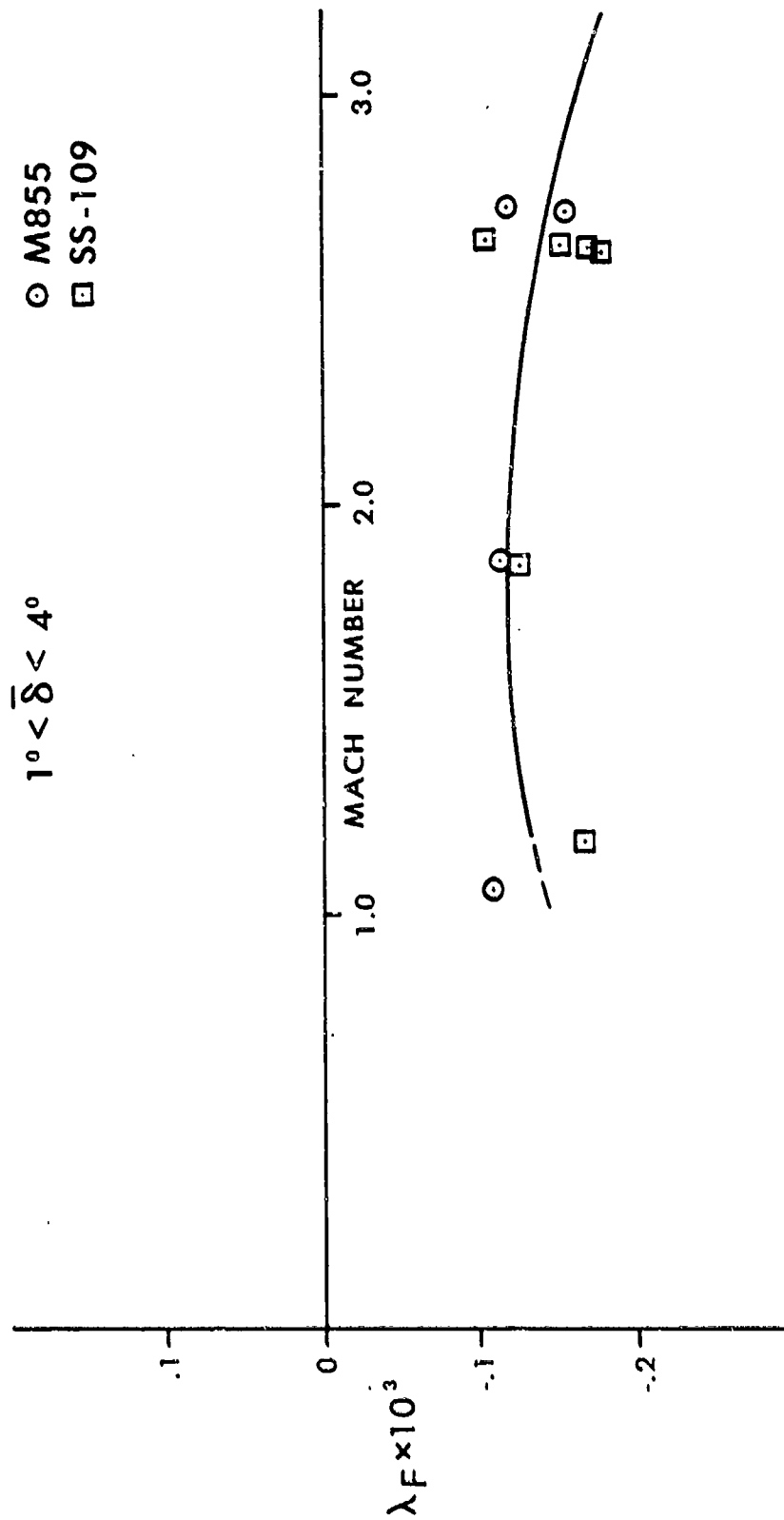


Figure 21. Fast Arm Damping Rate versus Mach Number, M855/SS-109

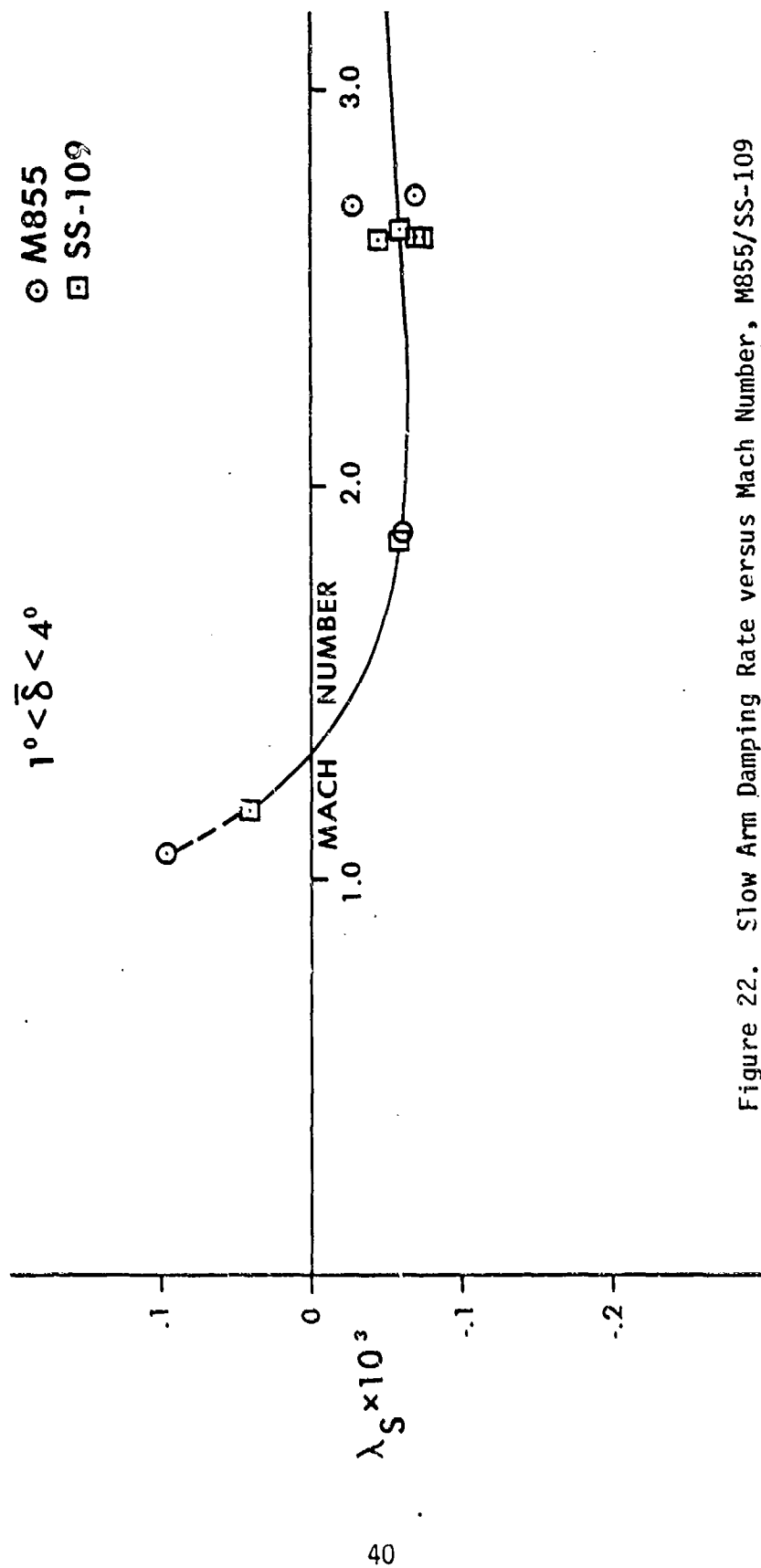


Figure 22. Slow Arm Damping Rate versus Mach Number, M855/SS-109

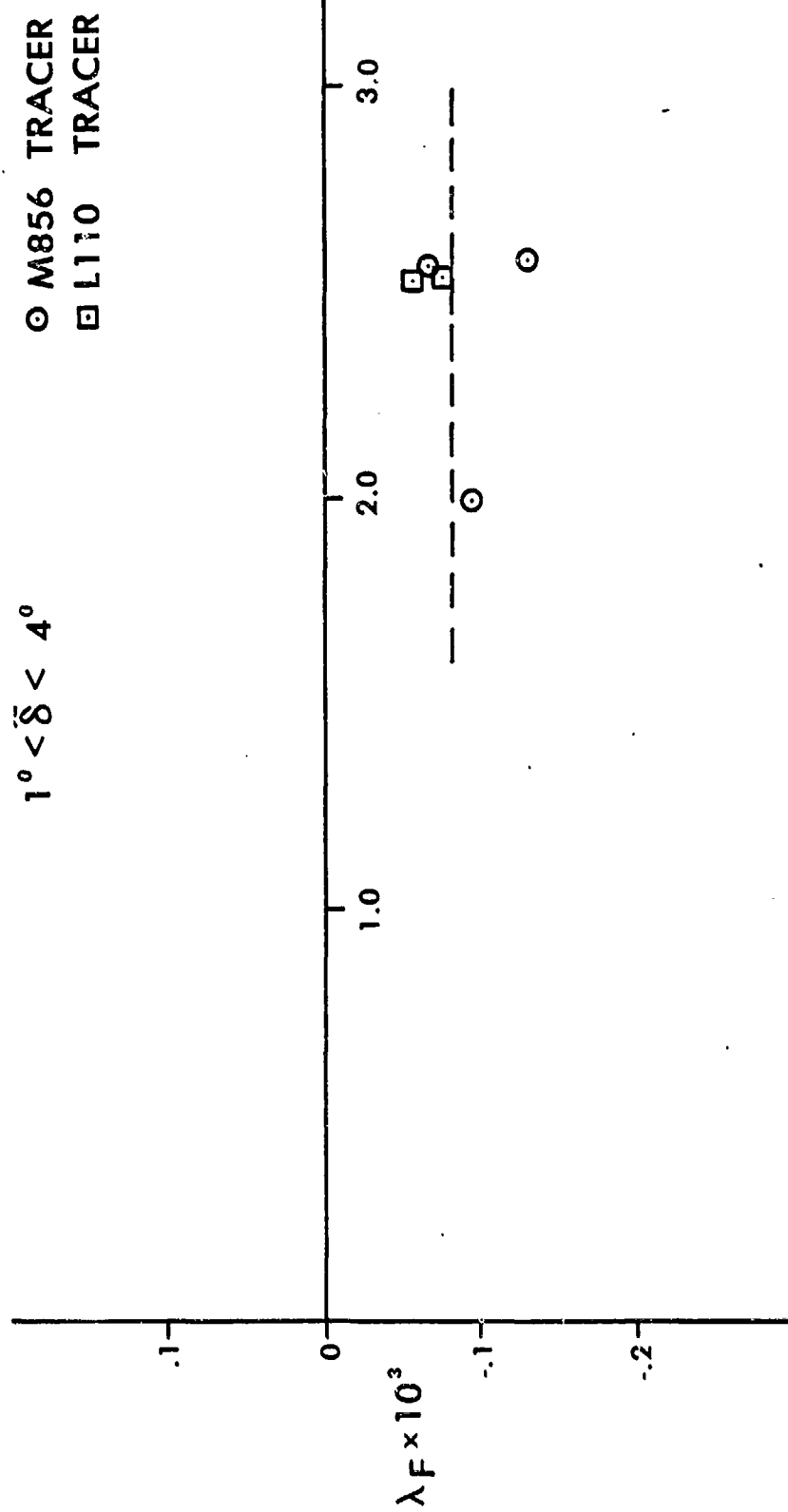


Figure 23. Fast Arm Damping Rate versus Mach Number, M856/L110

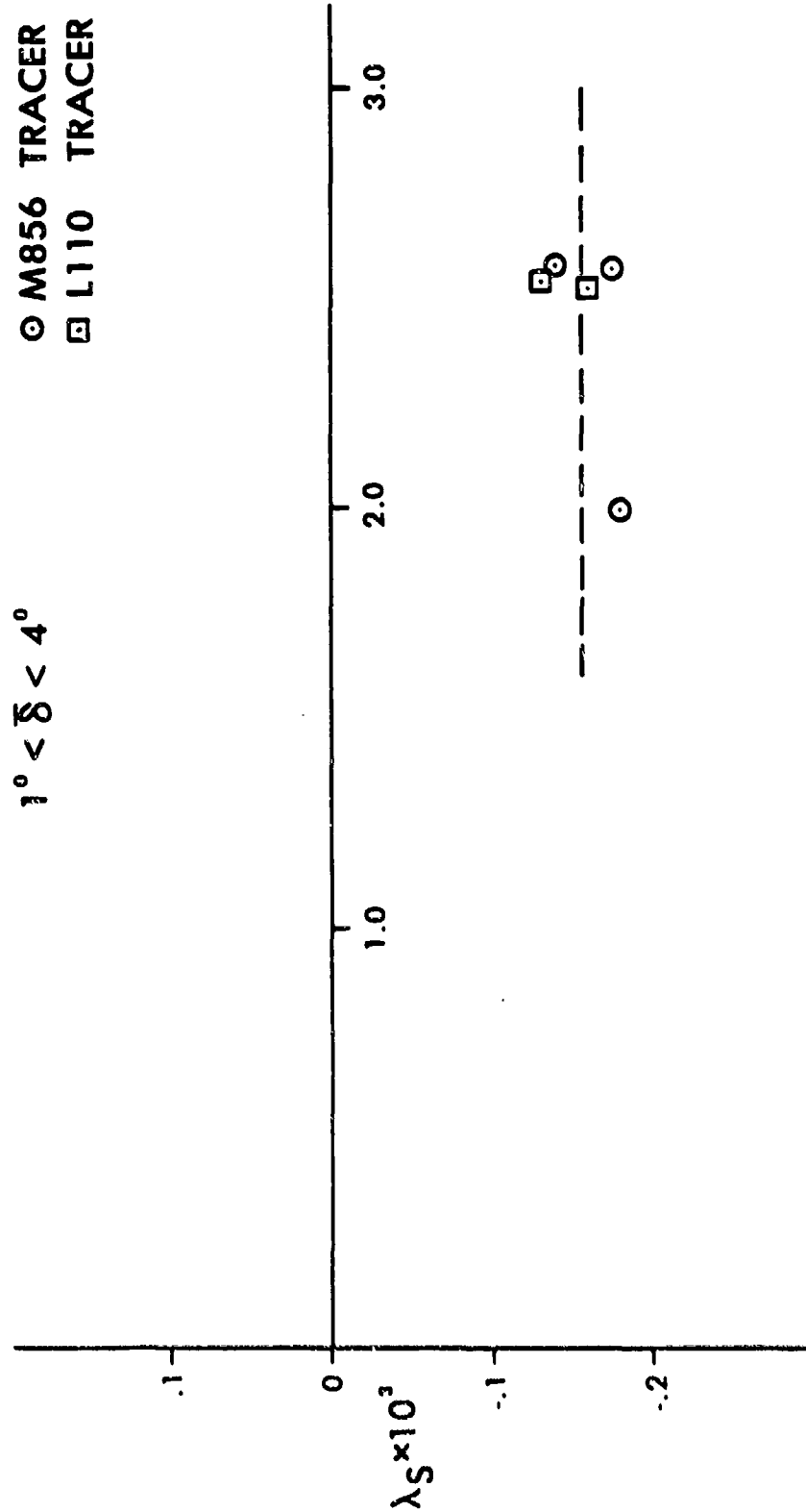


Figure 24. Slow Arm Damping Rate versus Mach Number, M856/L110

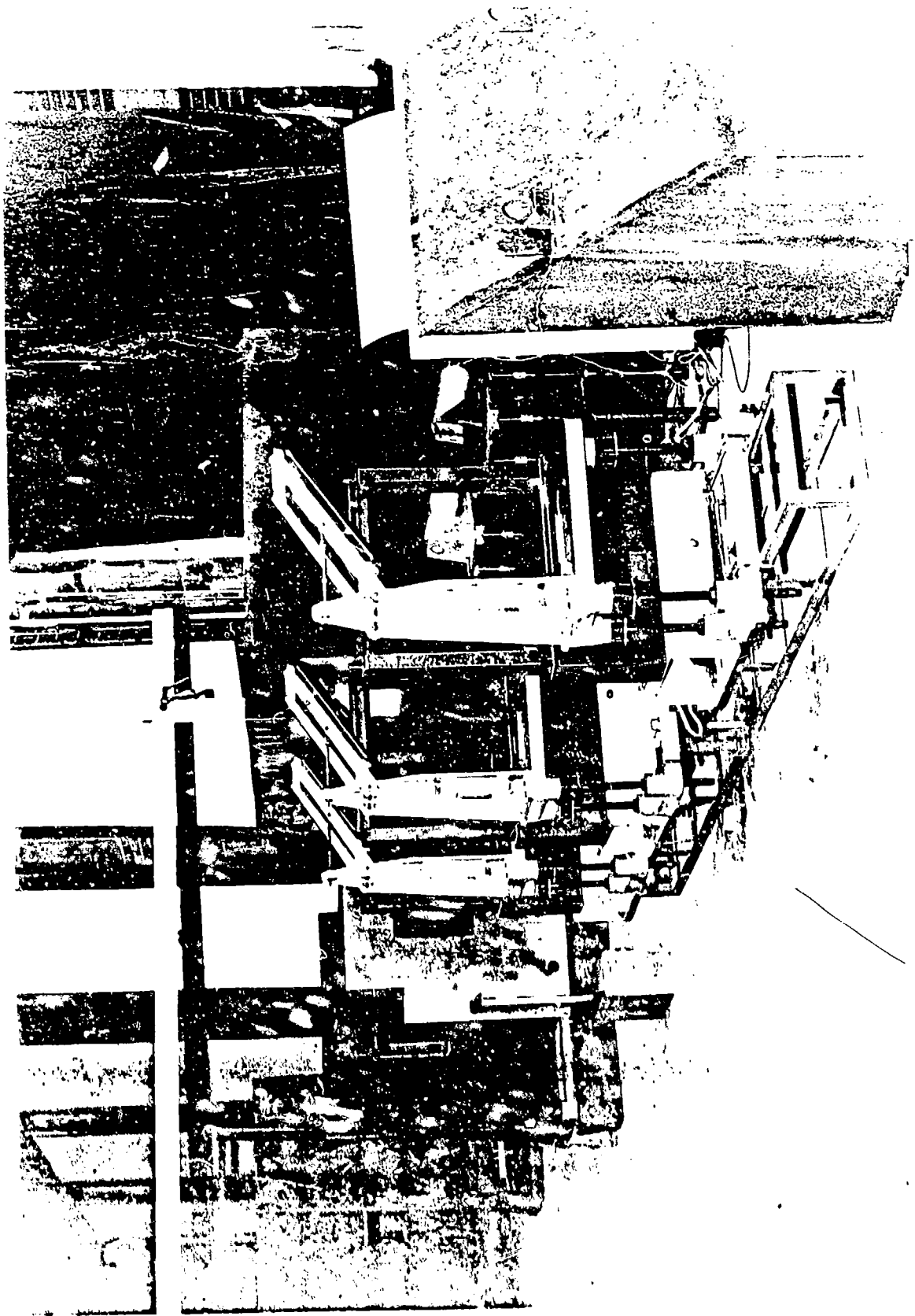


Figure 25. Photograph of Limit-Cycle Yaw Test Equipment

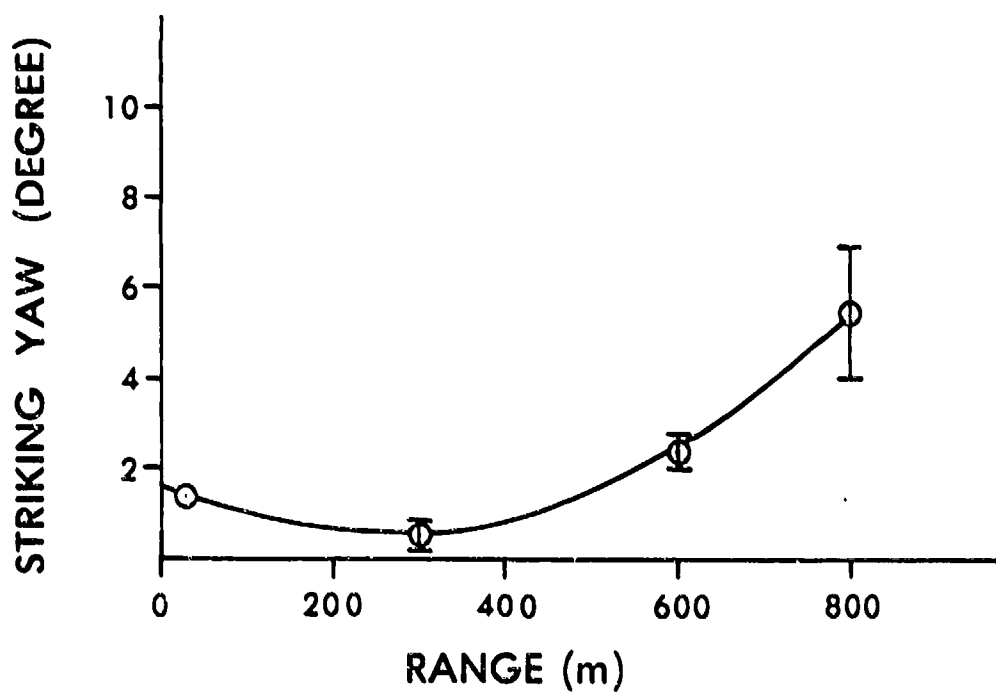
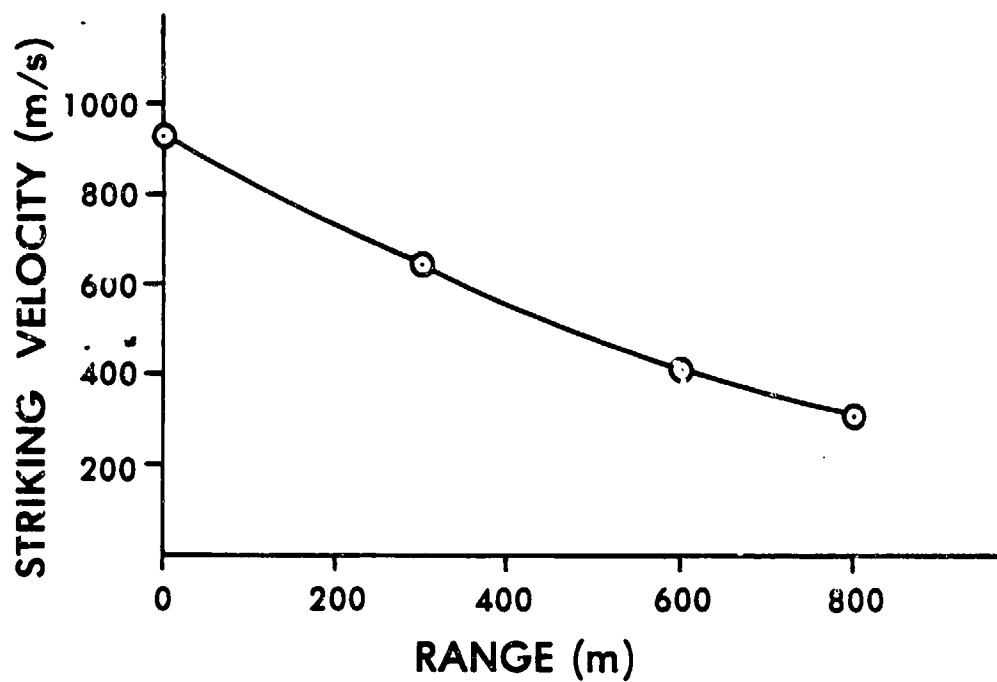


Figure 26. Striking Velocity and Striking Yaw versus Range, SS-109

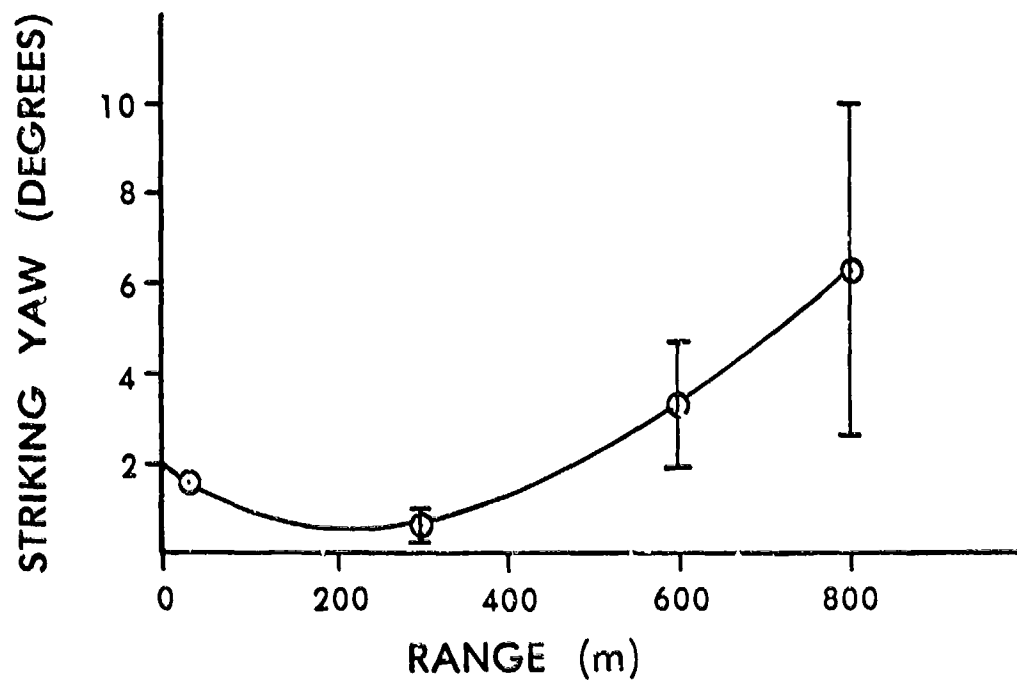
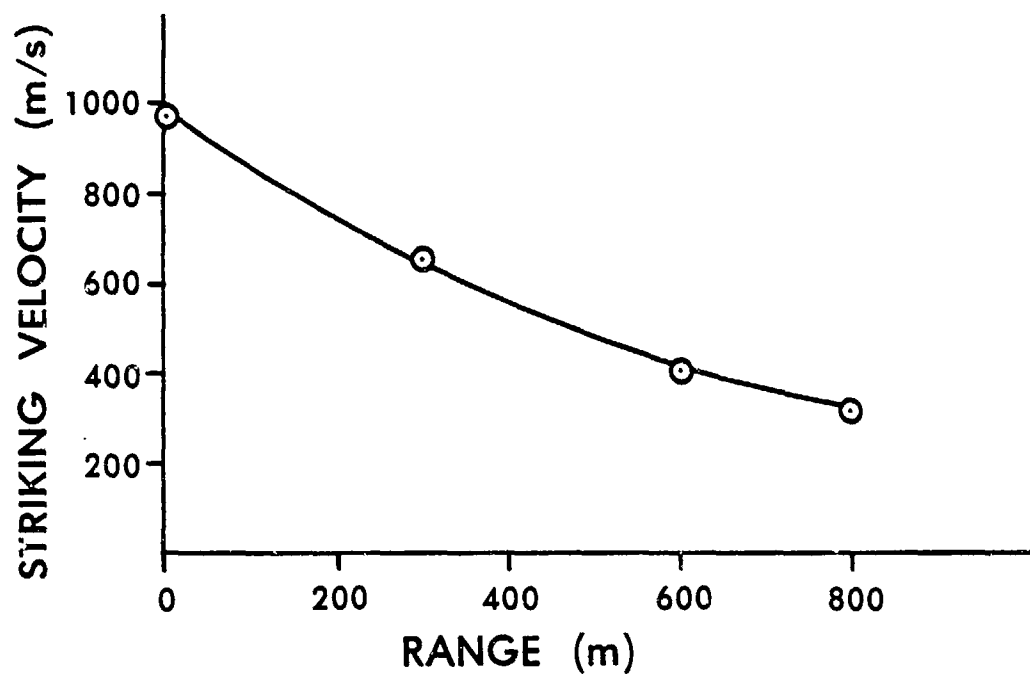


Figure 27. Striking Velocity and Striking Yaw versus Range, M855

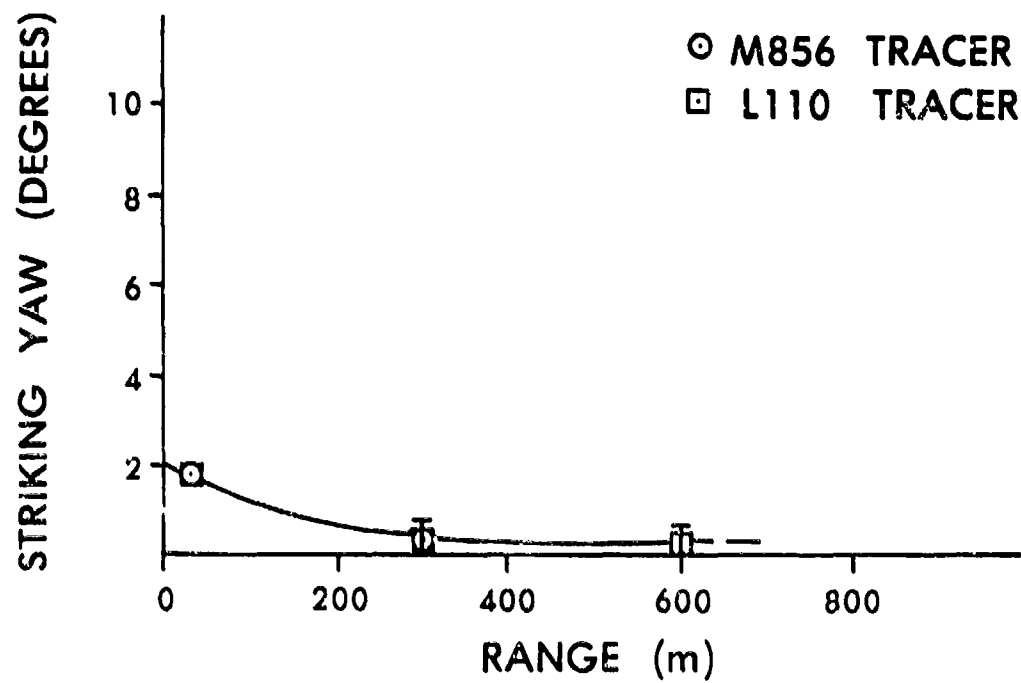
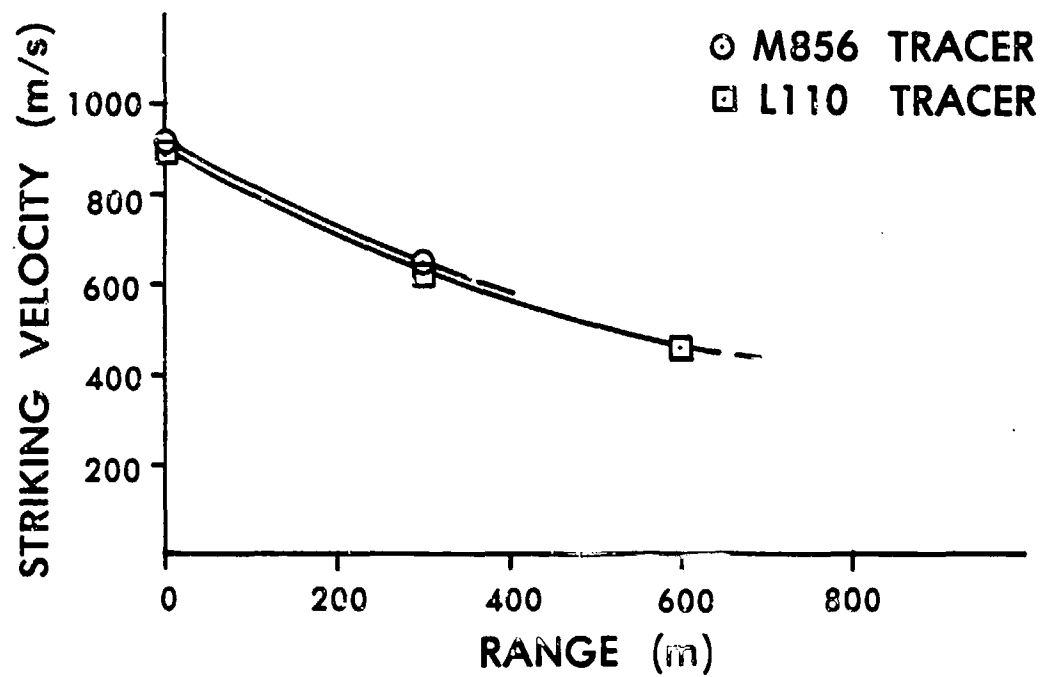


Figure 28. Striking Velocity and Striking Yaw versus Range, M856/L110

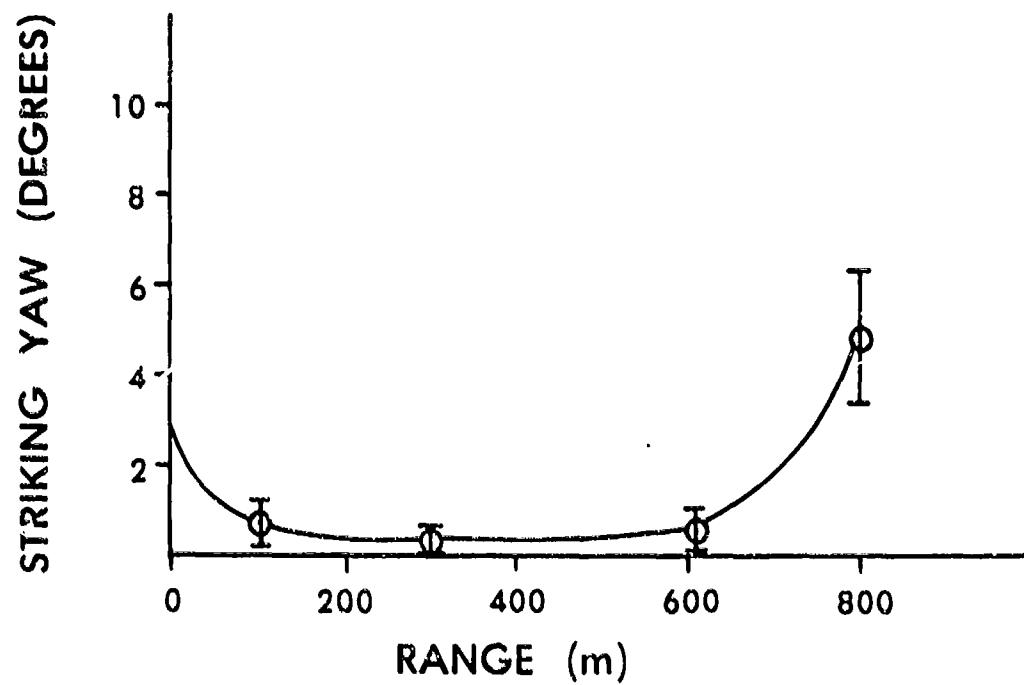
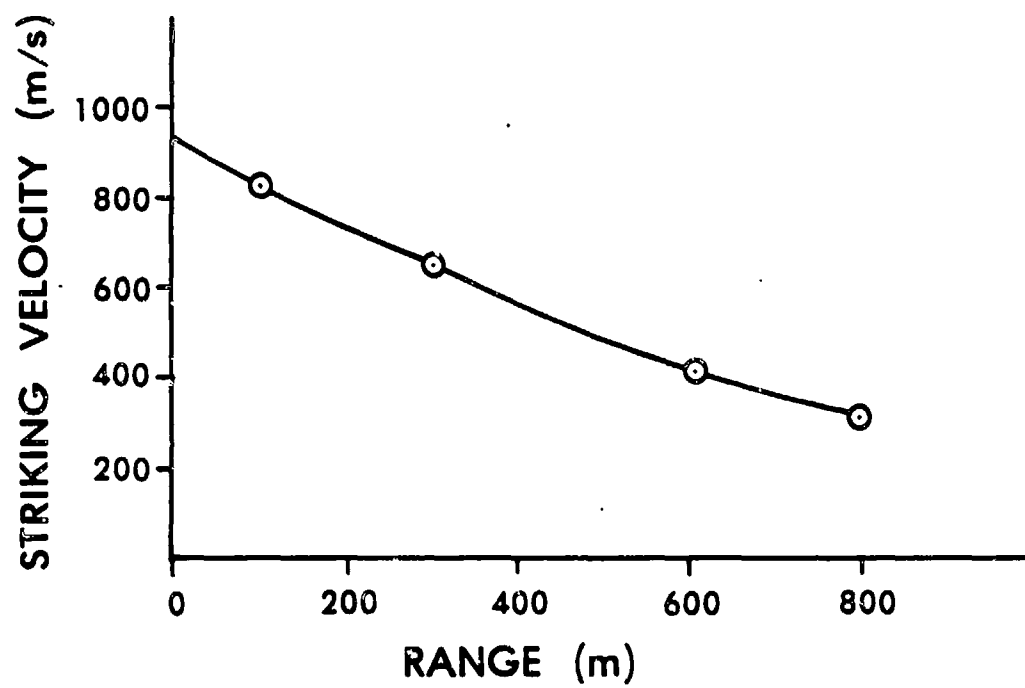


Figure 29. Striking Velocity and Striking Yaw versus Range, SS-109, from the XM249E1 Weapon

Table 1. Average Physical Characteristics of 5.56mm NATO Projectiles

Projectile	Reference Diameter (mm)	Weight (grams)	Center of Gravity (calibers, base)	Axial Moment of Inertia (gm-cm ²)	Transverse Moment of Inertia (gm-cm ²)
Ball, SS-109	5.69	4.03	1.52	.1425	1.112
Ball, M855	5.69	4.05	1.54	.1426	1.150
Tracer, L110	5.69	4.09	2.52	.1573	1.874
Tracer, M856	5.69	4.19	2.57	.1634	1.987

Table 2. Accuracy Firing Test Results

Date	Group No.	Kart Barrel No.	Cartridge Case	Primer	Propellant	Charge (Grains)	Bullet	Mean Radius (in. @ 100 yds.)
5/31/83	I	018	Federal	CCI 450	IMR 3031	25	52 SIERRA, HPBT	0.26
5/31/83	II	018	Federal	CCI 450	IMR 3031	25	52 SIERRA, HPBT	0.31
5/31/83	III	018	FNB	FNB	FNB	STD	SS109 (BALL)	0.61
5/31/83	IV	018	FNB	FNB	FNB	STD	L110 (TRACER)	1.40
6/1/83	V	014	Federal	CCI 450	IMR 3031	25	52 SIERRA, HPBT	0.19
6/1/83	VI	014	Federal	CCI 450	IMR 3031	25	52 SIERRA, HPBT	0.29
6/1/83	VII	014	FNB	FNB	FNB	STD	SS109 (BALL)	0.37
6/1/83	VIII	014	FNB	FNB	FNB	STD	L110 (TRACER)	1.23
6/1/83	IX	(OBERMEYER)	Federal	CCI 450	IMR 3031	25	52 SIERRA, HPBT	0.29
6/27/83	I	014	LC	LC	LC	STD	M855 (BALL)	1.32
6/27/83	II	014	FNB	FNB	FNB	STD	SS109 (BALL)	0.61
6/27/83	III	014	LC	LC	LC	STD	M855 (BALL)	0.93
6/27/83	*IV	014	LC	LC	LC	STD	M856 (TRACER)	1.30
6/27/83	V	014	Federal	CCI 450	IMR 3031	25	52 SIERRA, HPBT	0.21
6/27/83	VI	014	Federal	CCI 450	IMR 3031	23	M855 (BALL)	0.51
6/27/83	VII	014	Federal	CCI 450	LC	27	M855 (BALL)	0.86
6/27/83	VIII	014	LC	LC	IMR 3031	25	52 SIERRA, HPBT	0.20

*9 SHOT GROUP ---ONE ROUND WAS INADVERTENTLY NOT FIRED

Table 3 . Aerodynamic Coefficients of 5.56mm NATO Ball Projectiles

Rd. No.	Projectile Type	Mach Number	α_t (deg)	C_D	C_{M_α}	C_{L_α}	$C_{M_{p\alpha}}$	$(C_{M_q} + C_{M_{\dot{\alpha}}})$	CP_N (cal-base)
16221	SS-109	2.638	1.30	.2898	2.52	----	-.04	-3.77	----
16220		2.622	1.28	.2857	2.55	2.82	-.03	-6.42	2.34
16231		1.964	.88	.3346	2.76	----	----	----	----
16230		1.860	3.78	.3730	2.77	2.67	.04	-5.02	2.43
16238		1.179	2.90	.4614	2.97	2.15	-.31	-3.15	2.65
16239		1.138	1.47	.4488	----	----	----	----	----
16245(a)		.757	2.57	.1753	----	----	----	----	----
16245(b)		.746	4.06	.1832	----	1.81	----	----	----
16246		.736	4.82	.2204	3.05	1.61	-.16	-.22	3.19
*14414		2.645	.40	.2834	----	----	----	----	----
*14416		2.629	1.70	.2919	2.54	----	.16	-7.05	----
*14415		2.625	1.84	.2956	2.55	----	.13	-7.38	----
16222	M855	2.730	1.90	.3039	2.40	2.61	.10	-5.52	2.36
16223		2.714	1.22	.3007	2.35	----	-.03	-6.51	----
16228		1.875	1.62	.3741	2.66	----	----	----	----
16229		1.869	3.65	.3851	2.64	2.59	.06	-4.99	2.42
16237		1.137	2.27	.4874	2.82	2.16	----	----	2.60
16238		1.072	2.87	.4826	2.85	2.14	-.59	1.27	2.62
16243		.674	6.04	.2918	2.67	1.85	-.47	0.0	2.79

(a), (b) after a round number denotes a split reduction.

* Round previously fired from Obermeyer 7" twist barrel.

Table 4. Aerodynamic Coefficients of 5.56mm NATO Tracer Projectiles

Rd. No.	Projectile Type	Mach Number	α_t (deg)	C_D	C_{M_α}	C_{L_α}	$C_{M_{P\alpha}}$	$(C_{M_q} + C_{M_\alpha})$	CP_N (cal-base)
16224	L110	2.543	.78	.2983	2.44	----	.39	-10.4	----
16225		2.533	2.76	.3083	2.26	2.96	.44	-10.1	3.21
16234		1.890	6.14	.4267	2.50	2.89	.89	-18.6	3.27
16235		1.822	.52	.3737	----	----	----	----	----
16242		1.121	5.21	.5398	----	----	----	----	----
16249(a)		.854	4.31	.2470	----	----	----	----	----
16249(b)		.841	5.29	.2074	----	----	----	----	----
16226	M856	2.575	1.64	.3044	2.28	2.73	.45	-14.6	3.32
16227		2.566	1.70	.3140	2.29	2.81	.56	-12.5	3.30
16232		1.995	2.77	.3308	2.76	2.87	.59	-14.9	3.43
16233		1.870	4.01	.4025	2.70	----	.95	-19.8	----
16240(a)		1.184	4.03	.5031	2.88	2.10	----	----	3.67
16240(b)		1.140	5.95	.5334	2.86	2.11	----	----	3.65
16241		1.117	4.67	.4850	2.87	2.05	-.30	-5.15	3.70
16247		.758	7.59	.3174	2.93	1.92	----	----	3.87
16248		.738	10.04	.2847	----	----	----	----	----

(a), (b) after a round number denotes a split reduction.

Table 5. Flight Motion Parameters of 5.56mm NATO Ball Projectiles

Rd. No.	Projectile Type	Mach Number	S_g	S_d	$\lambda_F \times 10^3$ (1/cal)	$\lambda_S \times 10^3$ (1/cal)	K_F	K_S	ϕ'_F (rad/cal)	ϕ'_S (rad/cal)
16221	SS-109	2.638	2.53	.79	-.106	-.063	.0147	.0163	.0224	.0028
16220		2.622	2.56	.51	-.179	-.044	.0115	.0174	.0227	.0028
16231		1.964	2.39	---	---	---	.0048	.0142	.0227	.0031
16230		1.860	2.36	.73	-.125	-.060	.0376	.0497	.0225	.0031
16238		1.179	2.24	-.26	-.167	.040	.0160	.0468	.0225	.0033
16239		1.138	---	---	---	---	.0017	.0203	---	---
16245(a)		.757	---	---	---	---	.0071	.0403	---	---
16245(b)		.746	---	---	---	---	.0080	.0695	---	---
16246		.736	2.06	.19	-.036	.001	.0556	.0625	.0215	.0035
*14414		2.645	---	---	---	---	.0034	.0049	---	---
*14416		2.629	2.59	.74	-.154	-.077	.0134	.0204	.0229	.0028
*14415		2.625	2.54	.68	-.172	-.073	.0126	.0240	.0229	.0028
16222	M855	2.730	2.49	.81	-.118	-.072	.0207	.0232	.0213	.0027
16223		2.714	2.50	.44	-.155	-.027	.0107	.0173	.0211	.0027
16228		1.875	2.27	---	---	---	.0115	.0252	.0213	.0031
16229		1.869	2.29	.77	-.115	-.062	.0353	.0492	.0213	.0030
16237		1.137	2.27	---	---	---	.0063	.0366	.0218	.0032
16236		1.072	2.19	-11.6	-.109	.096	.0118	.0467	.0215	.0032
16243		.674	2.18	-2.87	-.113	.076	.0372	.0951	.0206	.0031

(a), (b) after a round number denotes a split reduction.

*Round previously fired from Obermeyer 7" twist barrel.

Table 6 . Flight Motion Parameters of 5.56mm NATO Tracer Projectiles

Rd. No.	Projectile Type	Mach Number	S_g	S_d	$\lambda_F \times 10^3$ (1/cal)	$\lambda_S \times 10^4$ (1/cal)	K_F	K_S	ϕ'_F (rad/cal)	ϕ'_S (rad/cal)
16224	L110	2.543	1.88	1.19	-.074	-.127	.0108	.0068	.0140	.0026
16225		2.533	2.06	1.34	-.055	-.158	.0399	.0219	.0143	.0024
16234		1.890	1.83	1.31	-.082	-.240	.0881	.0506	.0139	.0027
16235		1.822	----	----	----	----	.0049	.0061	----	----
16242		1.121	----	----	----	----	.0001	.0849	----	----
16249(a)		.854	----	----	----	----	.0104	.0722	----	----
16249(b)		.841	----	----	----	----	.0041	.0920	----	----
16226	M856	2.575	1.95	1.02	-.128	-.136	.0191	.0170	.0135	.0024
16227		2.566	1.91	1.32	-.064	-.173	.0239	.0137	.0133	.0024
16232		1.995	1.69	1.20	-.091	-.177	.0395	.0205	.0133	.0029
16233		1.370	1.66	1.33	-.081	-.257	.0597	.0225	.0130	.0029
16240(a)		1.184	1.57	----	----	----	.0107	.0654	.0128	.0032
16240(b)		1.140	1.64	----	----	----	.0089	.1018	.0132	.0030
16241		1.117	1.62	-.15	-.164	.041	.0347	.0704	.0131	.0031
16247		.758	1.49	----	----	----	.0847	.1008	.0123	.0033
16248		.738	----	----	----	----	.0029	.1622	----	----

(a), (b) after a round number denotes a split reduction.

Table 7. Limit-Cycle Yaw Test Results

Projectile Type	Range (M)	Total No. of stations Measured	Average Striking Yaw (Deg)	Standard Deviation in Yaw (Deg)	Average Striking Velocity (M/S)	Standard Deviation in Velocity (M/S)
SS-109	300	30	.50	.29	648.5	2.6
	600	30	2.41	.35	415.5	5.1
	800	57	5.54	1.43	315.1	3.1
M855	300	33	.55	.37	656.2	13.5
	600	41	3.29	1.39	406.2	8.4
	800	51	6.32	3.70	312.7	2.1
L110	300	39	.40	.33	638.0	5.3
	600	11	.35	.21	463.0	9.6
M856	300	28	.43	.22	644.2	15.8

Table 8. Dispersion Sensitivity Factors for 5.56mm Ammunition

Projectile	n (cal/turn)	$(k_t^2 - k_a^2)$	$\left(\frac{C_{L_\alpha}}{C_{M_\alpha}}\right)$	$\left[\left(\frac{2\pi}{n}\right) (k_t^2 - k_a^2) \left(\frac{C_{L_\alpha}}{C_{M_\alpha}}\right) \right]$
M193 Ball	53.6	.56	1.57	.10
M855 Ball	31.3	.77	1.13	.17
M196 Tracer	53.6	.75	1.83	.16
M856 Tracer	31.3	1.35	1.22	.33

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LIST OF SYMBOLS

$$C_D = \frac{\text{Drag Force}}{(1/2) \rho V^2 S}$$

$$C_{D_0} = \text{zero yaw drag coefficient}$$

$$C_{D_{\delta^2}} = \text{quadratic yaw drag coefficient}$$

$$C_{L_{\alpha}} = \frac{\text{Lift Force}}{(1/2) \rho V^2 S \delta}$$

Positive coefficient: Force in plane of total angle of attack, α_t , \perp to trajectory in direction of α_t . (α_t directed from trajectory to missile axis.) $\delta = \sin \alpha_t$.

$$C_{N_{\alpha}} = \frac{\text{Normal Force}}{(1/2) \rho V^2 S \delta}$$

Positive coefficient: Force in plane of total angle of attack, α_t , \perp to missile axis in direction of α_t .

$$C_{N_{\alpha}} \approx C_{L_{\alpha}} + C_D$$

$$C_{M_{\alpha}} = \frac{\text{Static Moment}}{(1/2) \rho V^2 S d \delta}$$

Positive coefficient: Moment increases angle of attack α_t .

$$C_{M_{p\alpha}} = \frac{\text{Magnus Moment}}{(1/2) \rho V^2 S d \left(\frac{pd}{V}\right) \delta}$$

Positive coefficient: Moment rotates nose \perp to plane of α_t in direction of spin.

$$C_{N_{p\alpha}} = \frac{\text{Magnus Force}}{(1/2) \rho V^2 S \left(\frac{pd}{V}\right) \delta}$$

Negative coefficient: Force acts in direction of 90° rotation of the positive lift force against spin.

LIST OF SYMBOLS (continued)

For most exterior ballistic uses, where $\dot{\alpha} = q$, $\dot{\beta} = -r$, the definition of the damping moment sum is equivalent to:

$$C_{M_q} + C_{M_{\dot{\alpha}}} = \frac{\text{Damping Moment}}{(1/2) \rho V^2 S d \left(\frac{q_t d}{V}\right)} \quad \text{Positive coefficient: Moment increases angular velocity.}$$

$$C_{\ell_p} = \frac{\text{Roll Damping Moment}}{(1/2) \rho V^2 S d \left(\frac{p d}{V}\right)} \quad \text{Negative coefficient: Moment decreases rotational velocity.}$$

CP_N = center of pressure of the normal force, positive from base to nose

α, β = angle of attack, side slip

$$\alpha_t = (\alpha^2 + \beta^2)^{1/2} = \sin^{-1} \delta, \text{ total angle of attack}$$

λ_F = fast mode damping rate
 λ_S = slow mode damping rate
 } negative λ indicates damping

ρ = air density

ϕ'_F = fast mode frequency

ϕ'_S = slow mode frequency

CG = center of gravity

d = body diameter of projectile, reference length

I_x = axial moment of inertia

I_y = transverse moment of inertia

LIST OF SYMBOLS (continued)

k_a^2	=	$I_x/m d^2$
k_t^2	=	$I_y/m d^2$
K_F	=	magnitude of the fast yaw mode
K_S	=	magnitude of the slow yaw mode
l	=	length of projectile
m	=	mass of projectile
M	=	Mach number
p	=	roll rate
q, r	=	transverse angular velocities
q_t	=	$(q^2 + r^2)^{1/2}$
R	=	subscript denotes range value
s	=	dimensionless arc length along the trajectory
S	=	$\frac{\pi d^2}{4}$, reference area
S_d	=	dynamic stability factor
S_g	=	gyroscopic stability factor
V	=	velocity of projectile

Effective Squared Yaw Parameter

$$\tilde{\delta}^2 = K_F^2 + K_S^2$$

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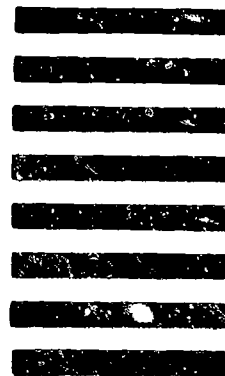
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